Integrated Systems for Marine Environment

# A Glimpse at the H2020 WiMUST project and one of its bits: outlier robust state estimation in marine robotics applications

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Università del Salento, Lecce - ISME node

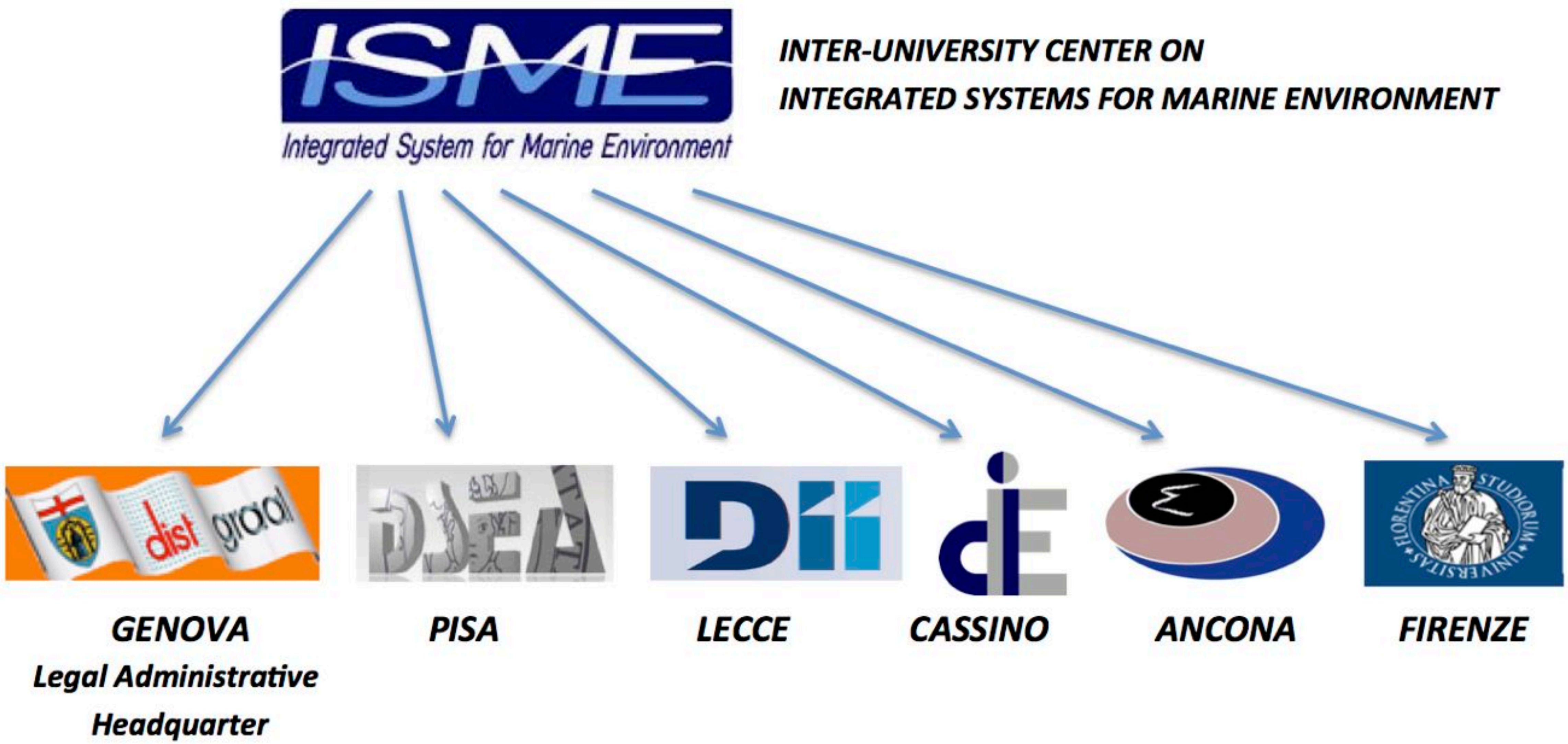
Napoli, 21st April 2015

# Outline

- The inter-university center ISME (Integrated Systems for the Marine Environment): an overview
- WiMUST: Widely scalable Mobile Underwater Sonar Technology. An H2020 project
- Underwater vehicle navigation and outlier robust state estimation

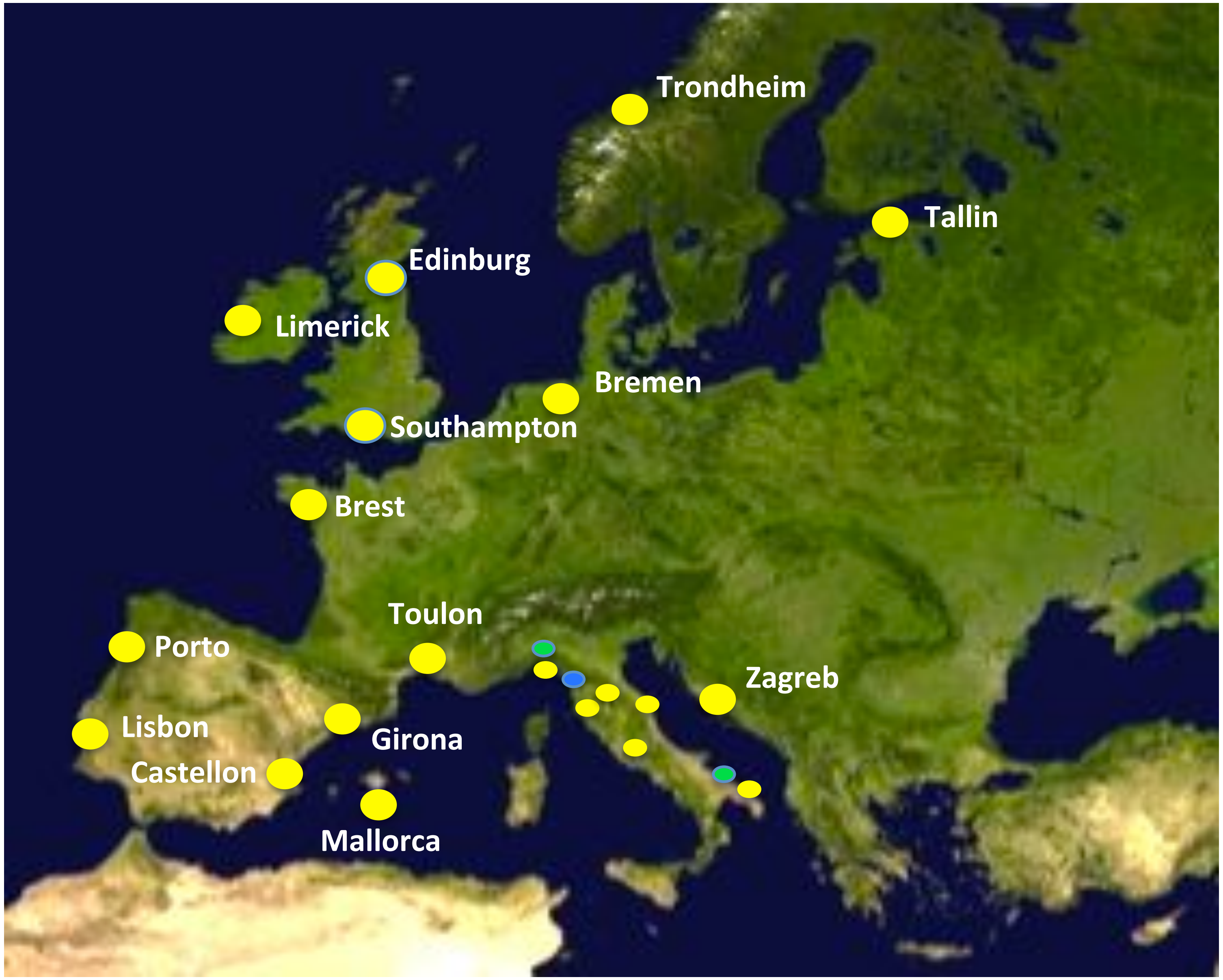
# The ISME Network

## *ISME Membership*



# Marine Robotics in Italy


- ISME
- CNR-ISSIA
- NATO-CMRE




**Trondheim** — NTNU

**Edinburg** — Heriot Watt University

**Tallin** — University of Tallin

**Limerick** — University of Limerick

**Bremen** — Jacobs University

**Toulon** — Ifremer

**Lisbon** — Istituto Superiore tecnico

**Girona** — University of Girona

**Zagreb** — University of Zagreb

**Castellon** — Universitat Jaume Primero

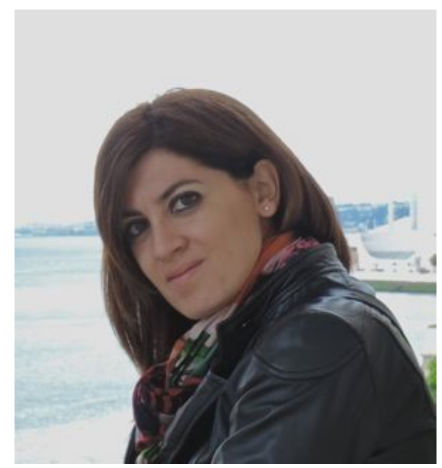
**Mallorca** — Universitat De Illes Balears

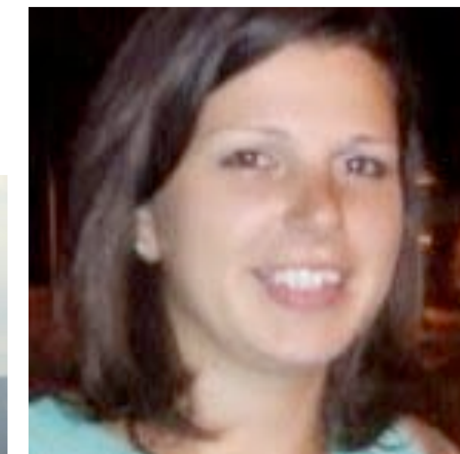
# The ISME Crew









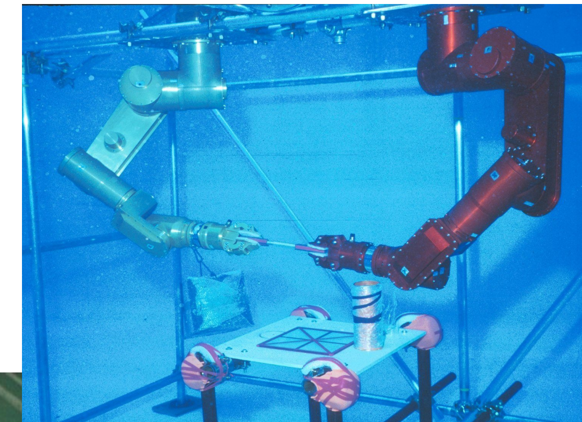


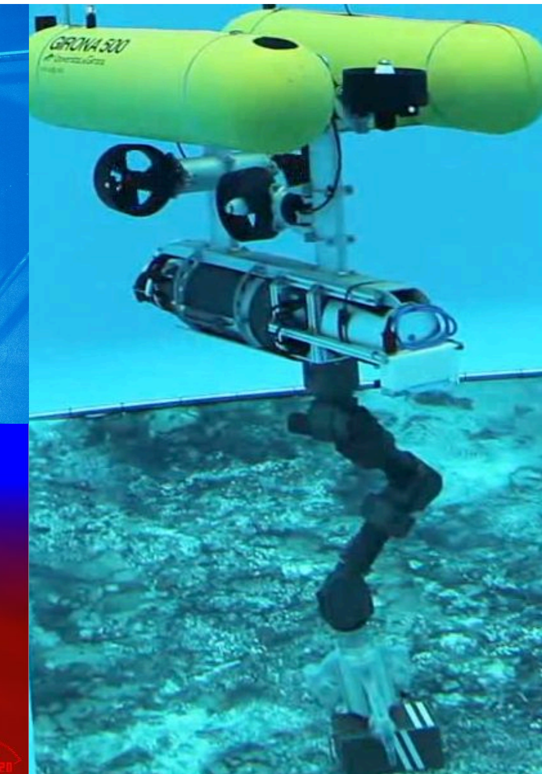


# ISME Funding and Resources

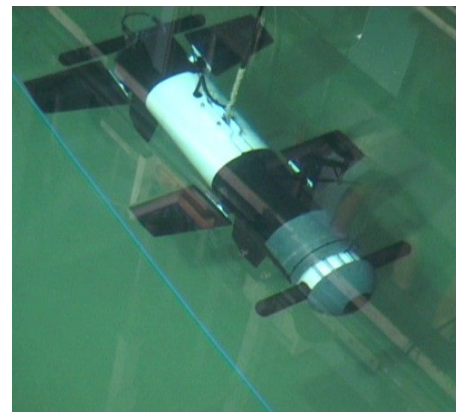
- ISME does not receive any institutional funding for its operation
- All ISME research is funded by contracts with agencies, industries, third parties
- Resources from any participating lab are available to ISME
- ISME-owned resources are available to any participating lab
- Total Human resources available: more than 35 structured researchers and more than 15 non-structured young researchers
- Strong emphasis is given to applied research and field activities
- Pointing toward unifying frameworks encompassing most of the applications is constantly encouraged
- Average Budget per year: 550 K-Euro (last 5 years average)

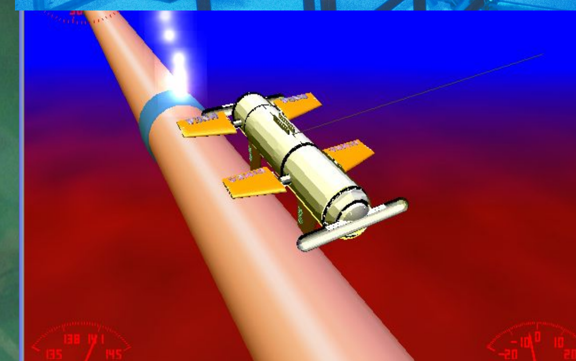
- Robotics
  - Underwater manipulation systems
  - Guidance and control of AUV's and ROV's
  - Distributed coordination and control of AUV's team
  - Mission planning and control



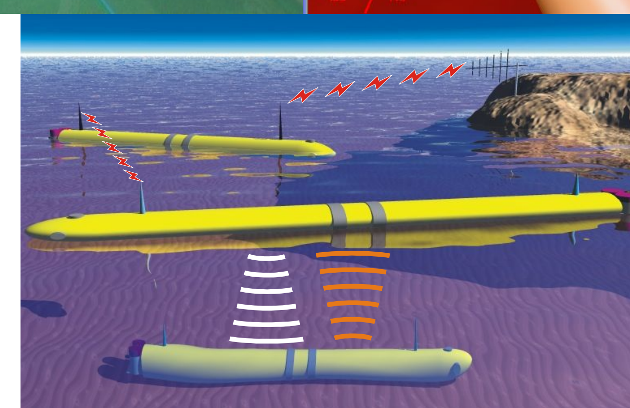


- Underwater acoustics
  - Acoustic localization
  - Acoustic communications
  - Underwater optical communications,
  - Acoustic Imaging and Tomography
  - Seafloor acoustics
  - Sonar systems

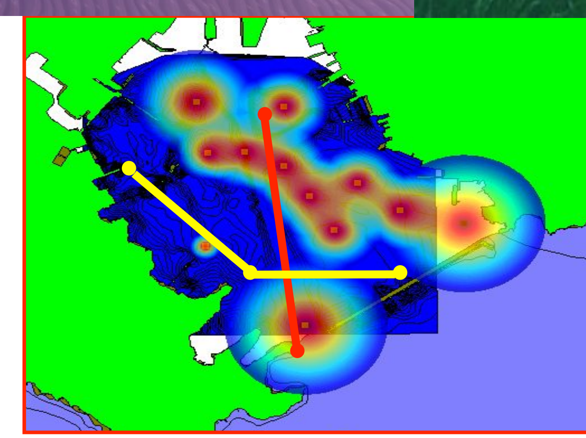


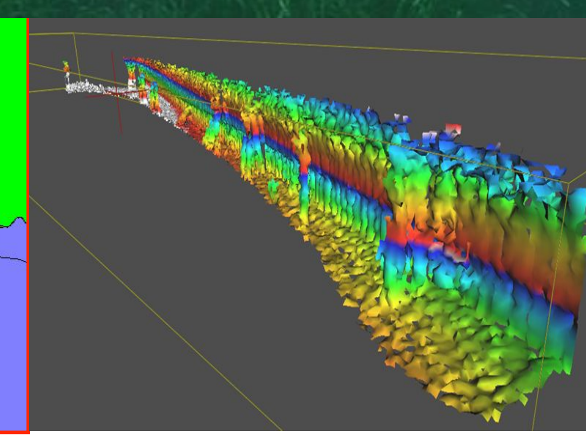


- Signal Processing and data acquisition
  - Distributed data acquisition
  - Geographical information systems
  - Decision support systems
  - Classification and data fusion



- Applications:
  - Surface and underwater security systems
  - Distributed underwater environmental monitoring
  - Underwater archaeology
  - Underwater infrastructures inspection
  - Sea surface remote sensing





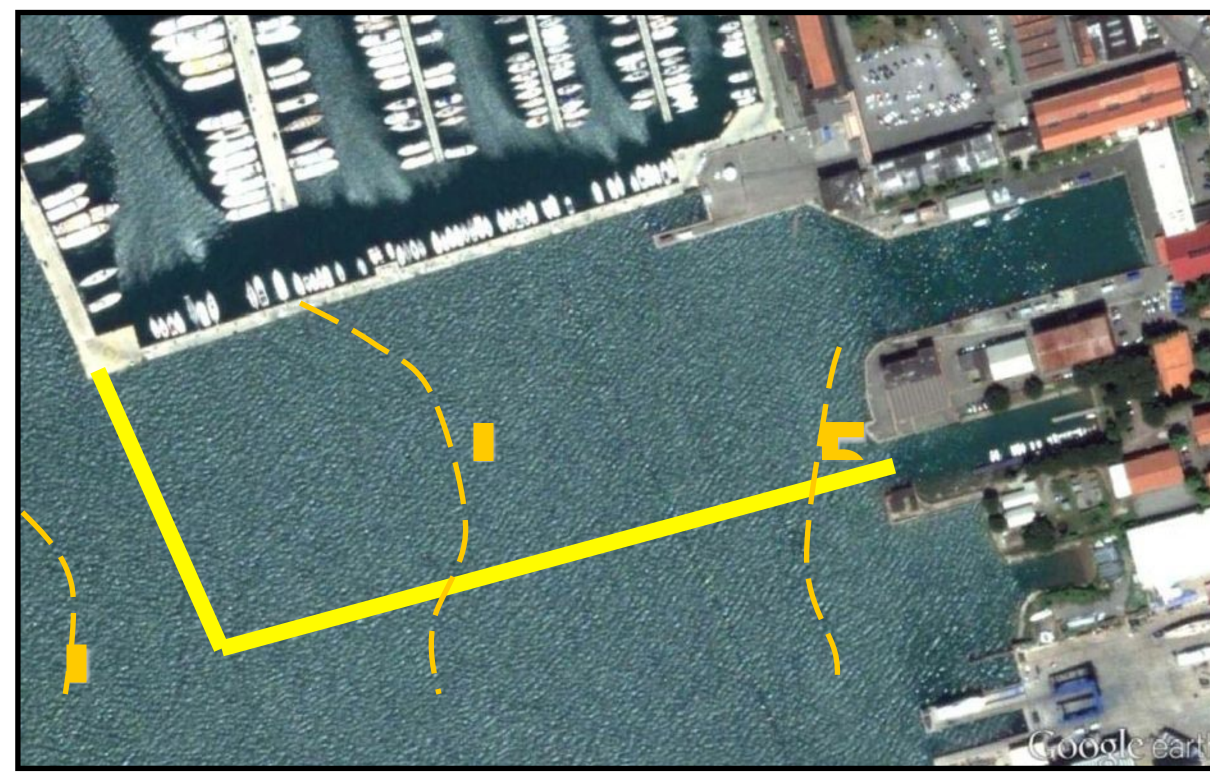
CENTRO SUPPORTO CSSN SPERIMENTAZIONE NAVALE

SEA-Lab

ISME Integrated Systems for Marine Environment

Laboratorio applicato per Sistemi Eterogenei Autonomi Marini

SISTEMI ETEROGENEI AUTONOMI - LABORATORIO



WiMUST – Widely scalable Mobile Underwater Sonar Technology
Grant agreement no: 645141

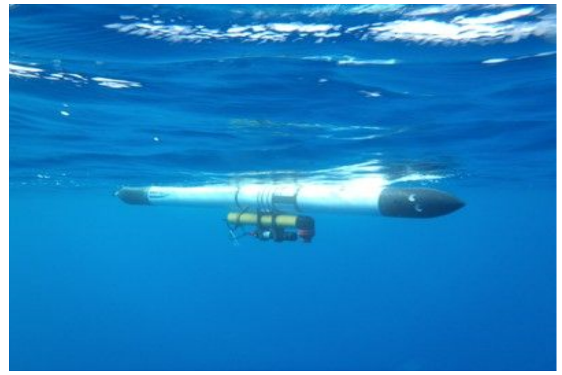
H2020 ICT-23-2014: Robotics
Started on February 1st, 2015
Duration 36 months
Maximum grant amount is EUR 3,970,081.25

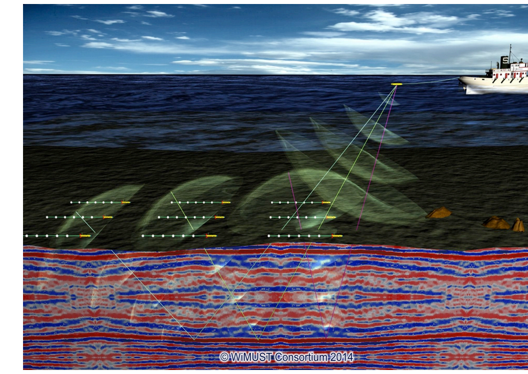
# Action Overview

- ISME (UNIVERSITA DEGLI STUDI DI GENOVA) – IT
- ASSOCIACAO DO INSTITUTO SUPERIOR TECNICO PARA A INVESTIGACAO E DESENVOLVIMENTO – PT
- CINTAL – CENTRO INVESTIGACAO TECNOLOGICA DO ALGARVE – PT
- THE UNIVERSITY OF HERTFORDSHIRE HIGHER EDUCATION CORPORATION – UK
- EVOLOGICS GMBH – DE
- GRAAL TECH SRL – IT
- CGGVERITAS SERVICES SA – FR
- GEO MARINE SURVEY SYSTEMS BV – NL
- GEOSURVEYS – CONSULTORES EM GEOFISICA LDA – PT

# Which Step Change will our project achieve in robotics technology and ability?

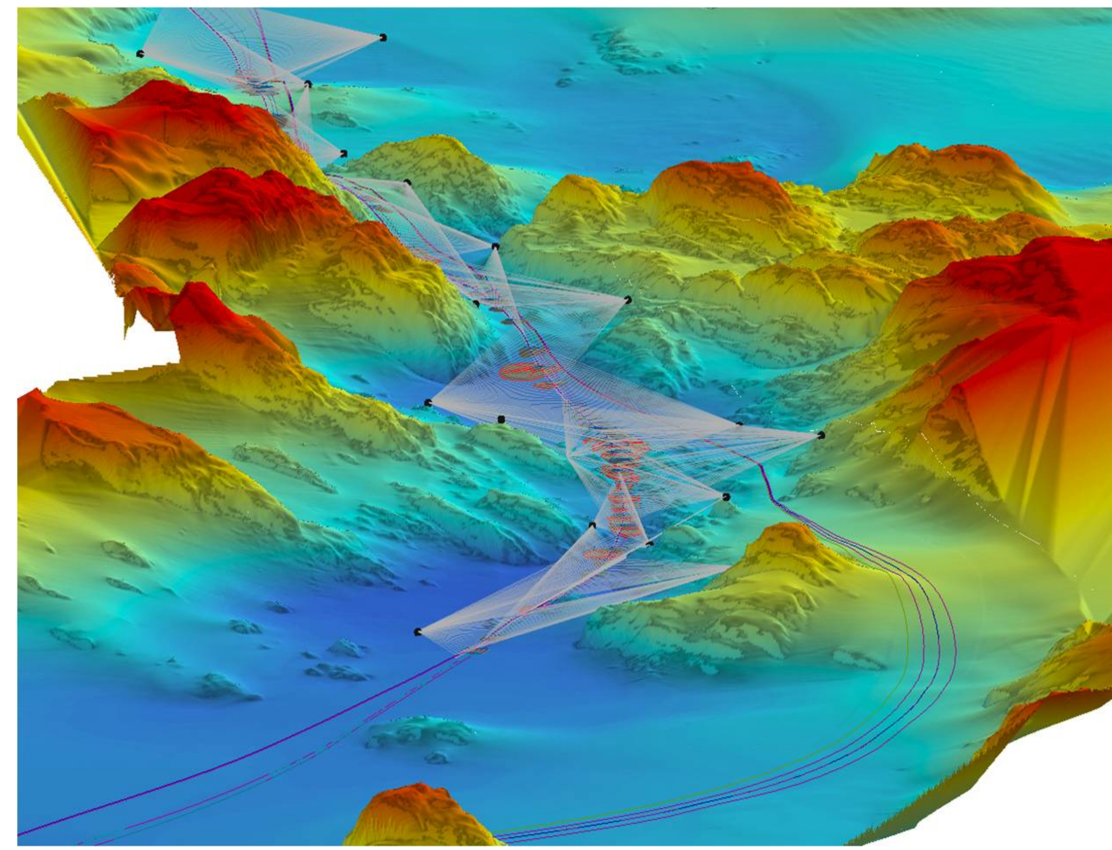
**WiMUST**
Widely scalable Mobile Underwater Sonar Technology





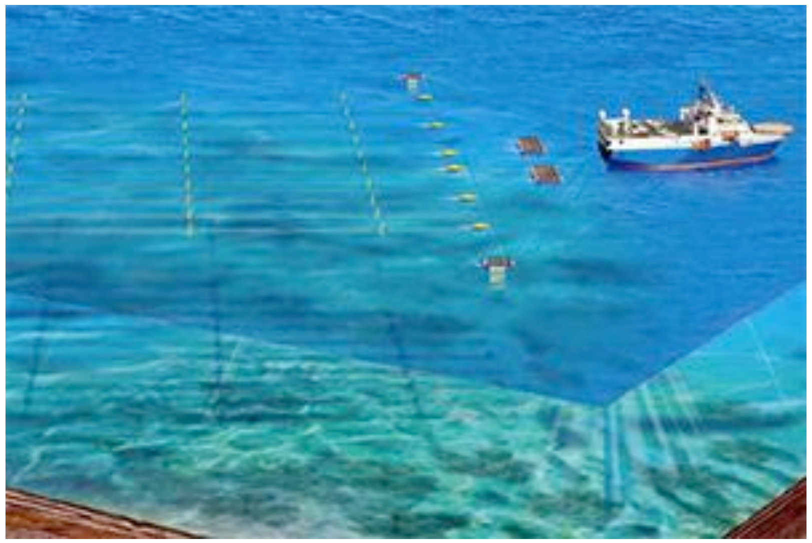
Market domain:
Marine Robotics (Civil & Commercial)

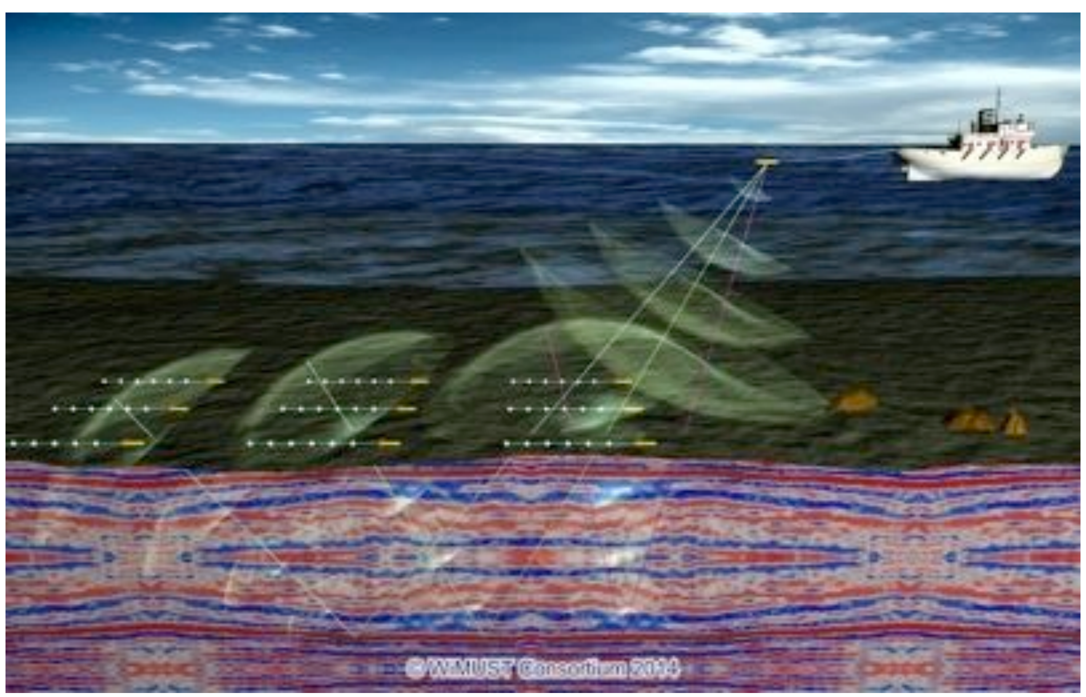
- Geotechnical Surveying
- Distributed Sensor Array
- Geophysical Surveying
- Monitoring

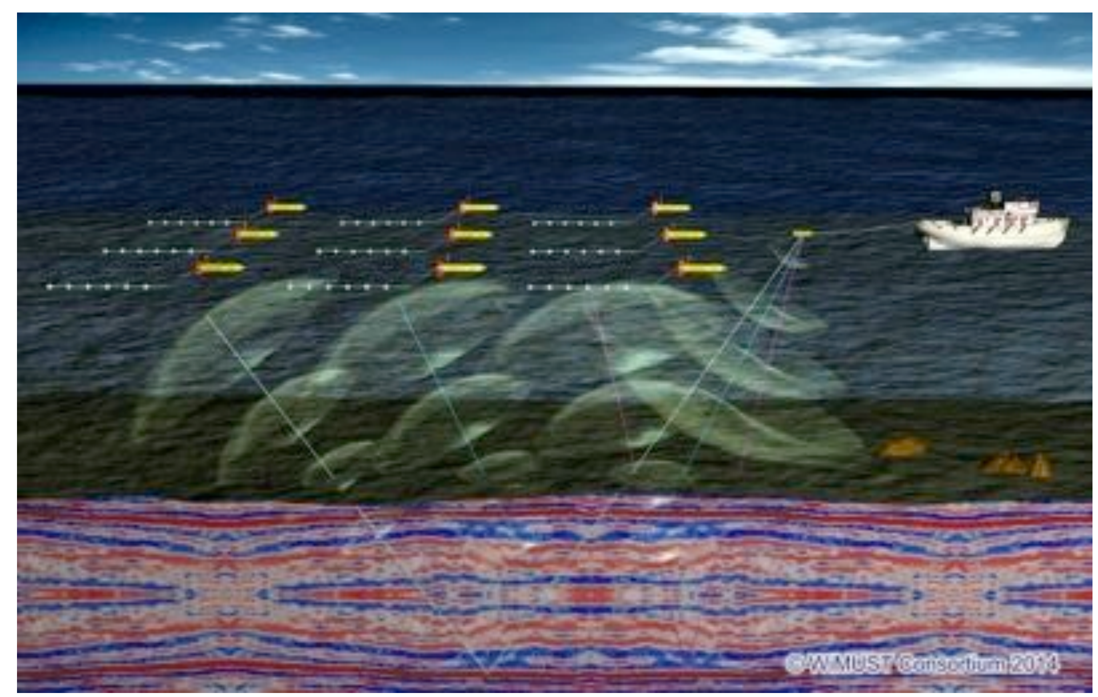


Achieve reliable cooperative operations of 10 or more marine robots for a demanding challenge in terms of navigation, guidance, control, mission planning, acoustic communications and data acquisition for geotechincal applications.

# How: what is our approach?



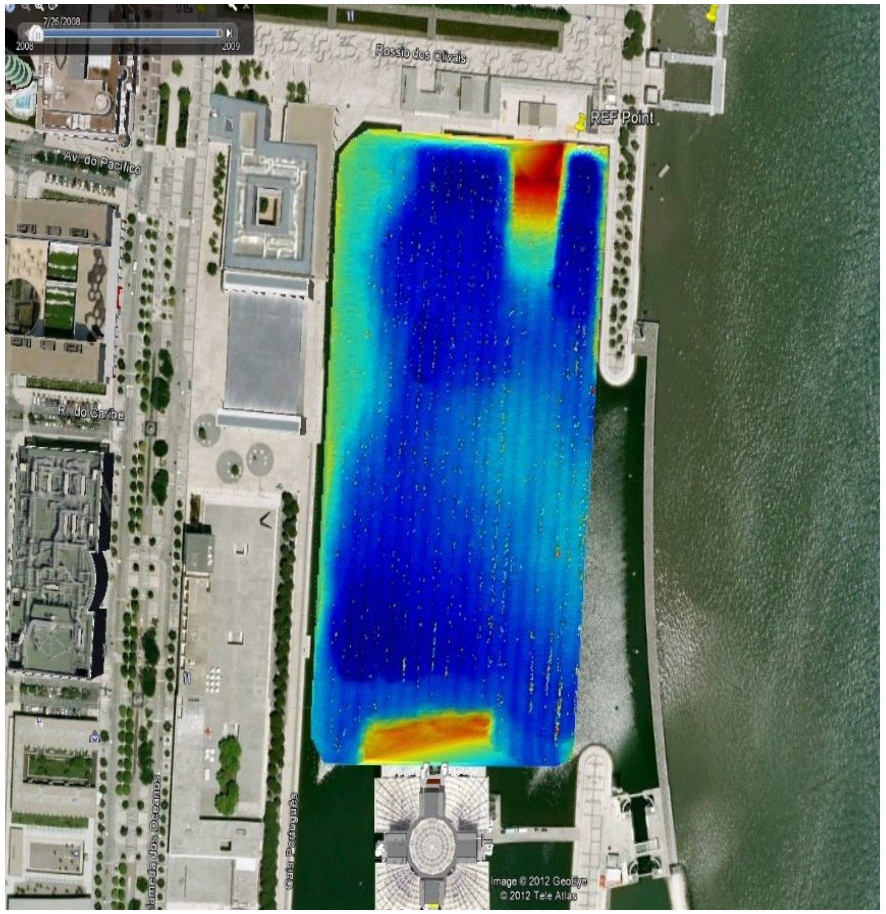


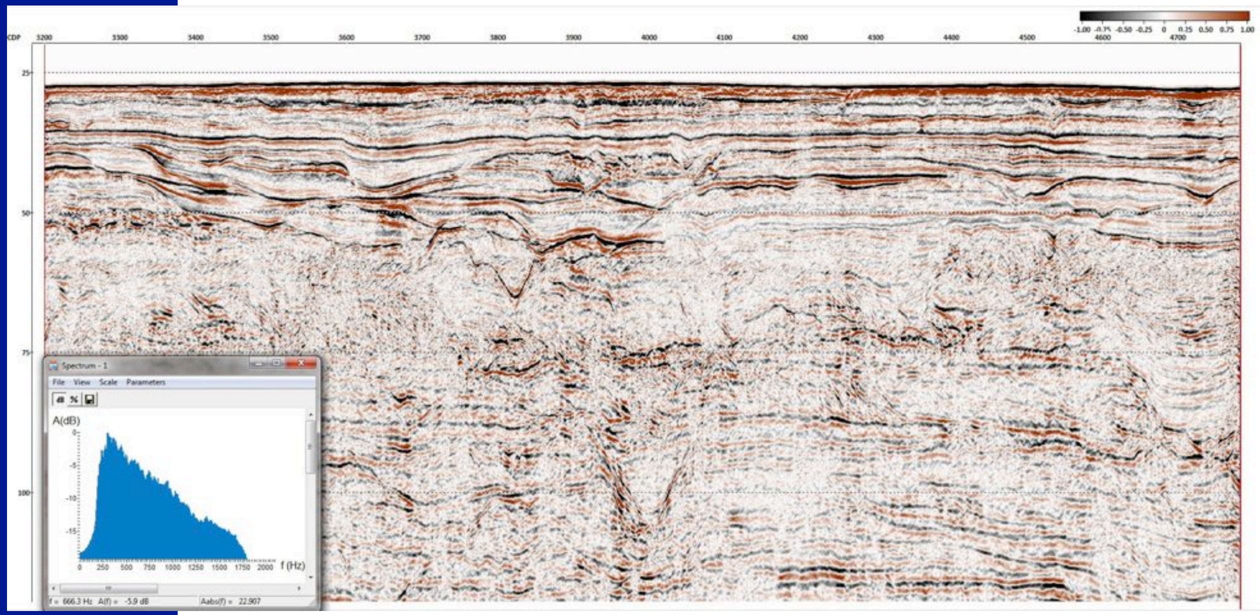
# Main challenges

- Acoustic Distributed Sensor Array;
- Communications (short and long range);
- Geotechnical surveying and Geophysical characterization;
- Clock synchronization (below 50 mu sec);
- Cooperative Navigation and Motion Control: accurate formation control;
- HW integration of the acoustic acquisition system with the navigation one;

# Validation Plans



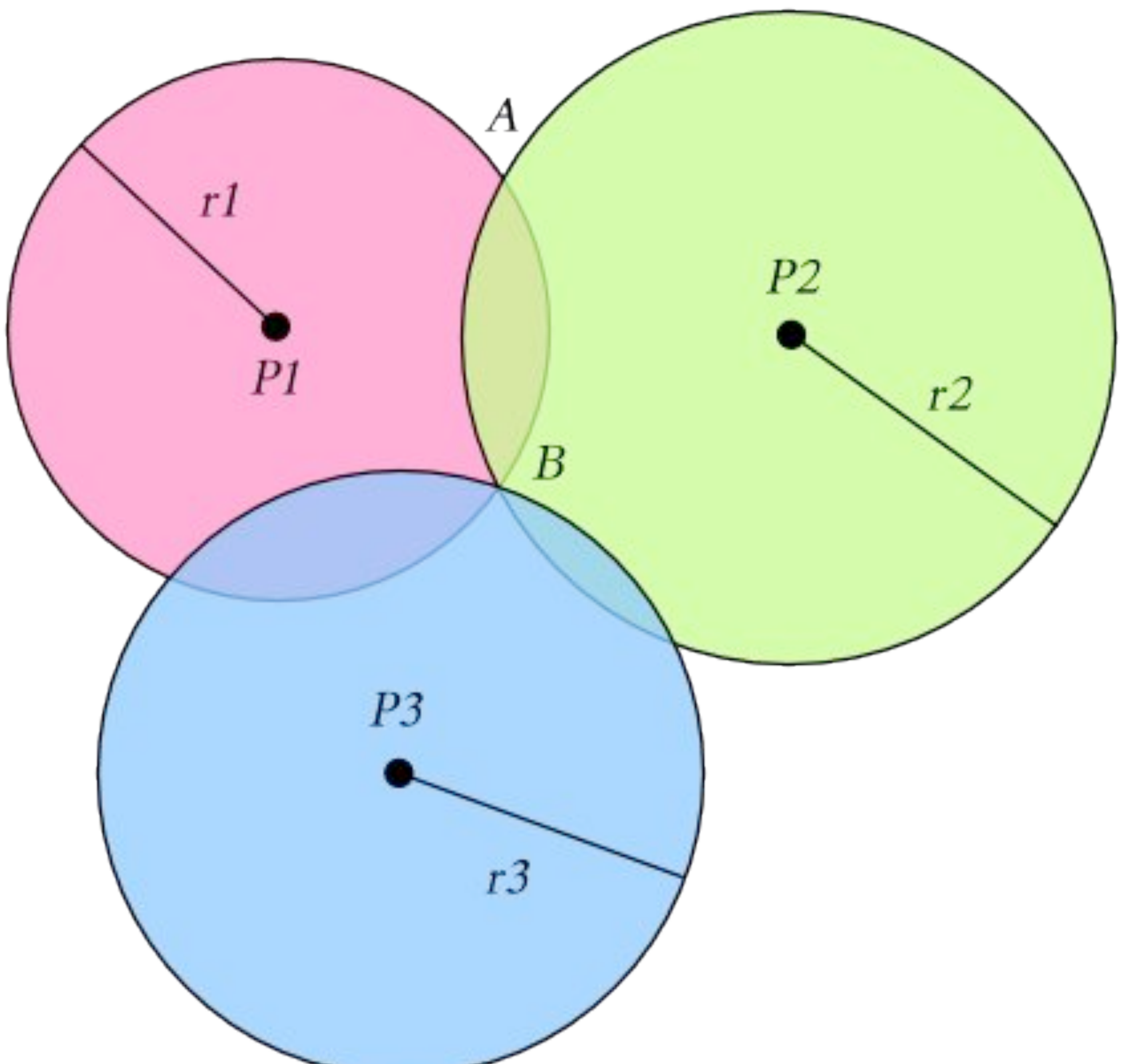


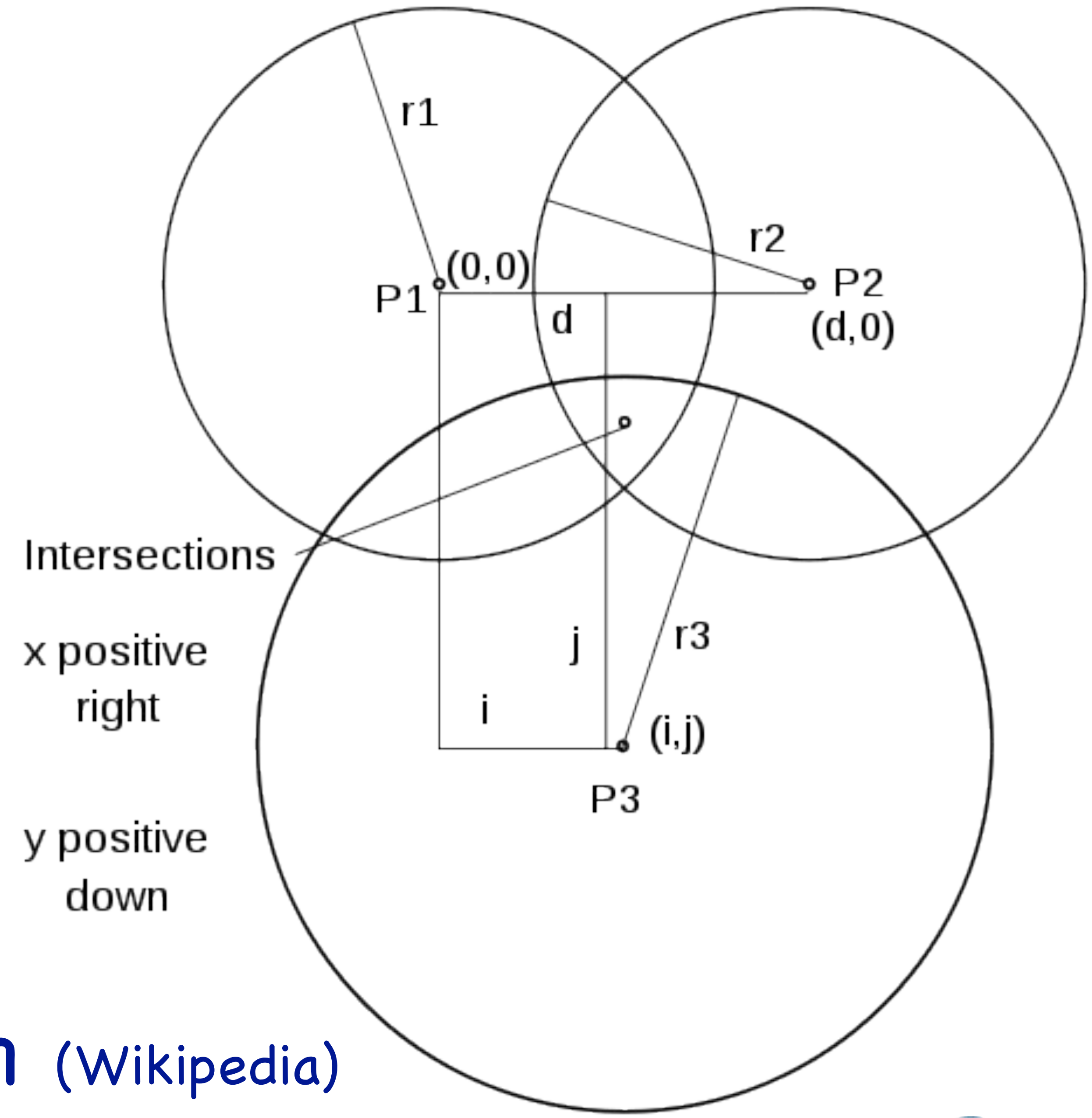





European Commission

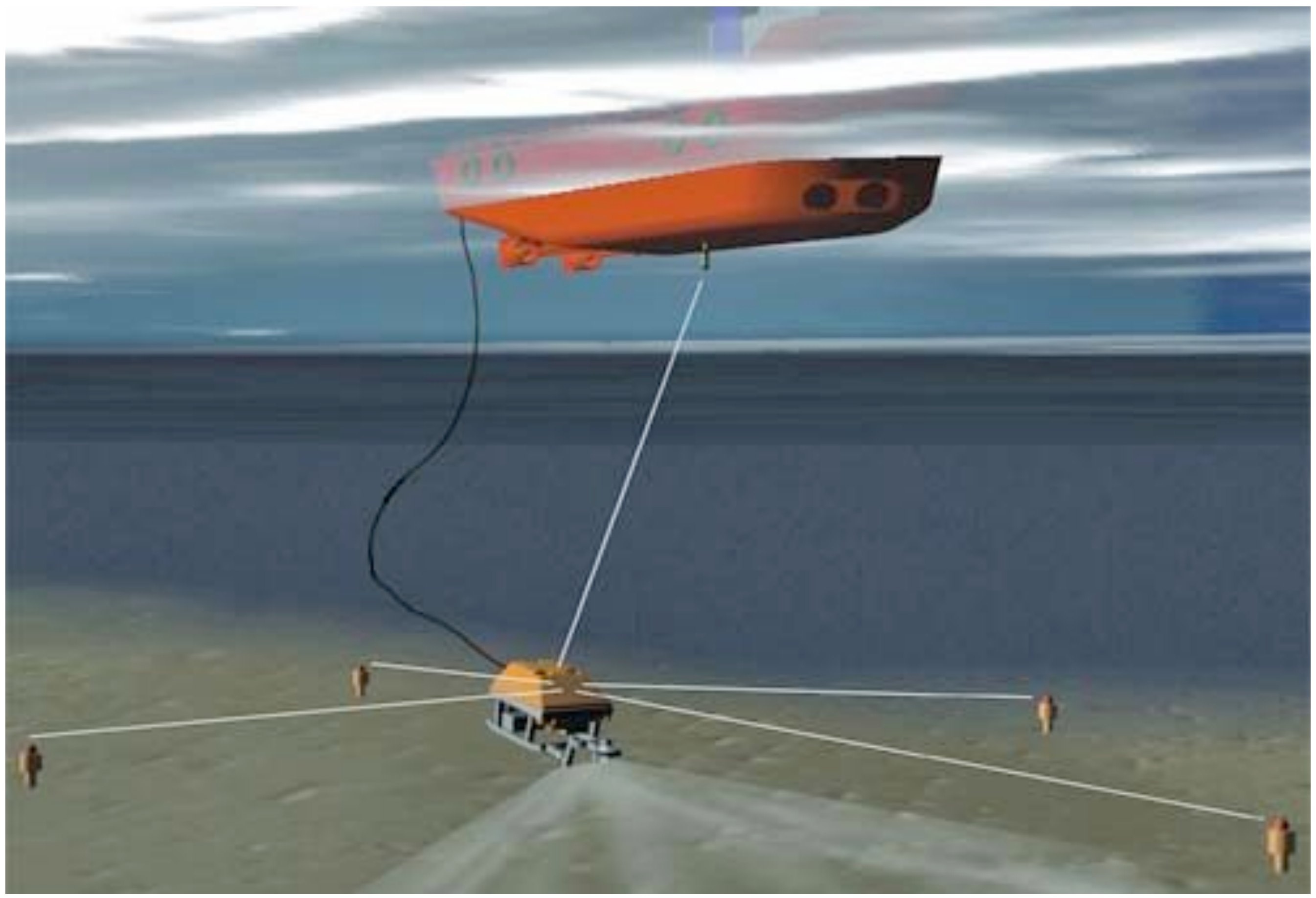
# Cooperative Navigation: single range navigation



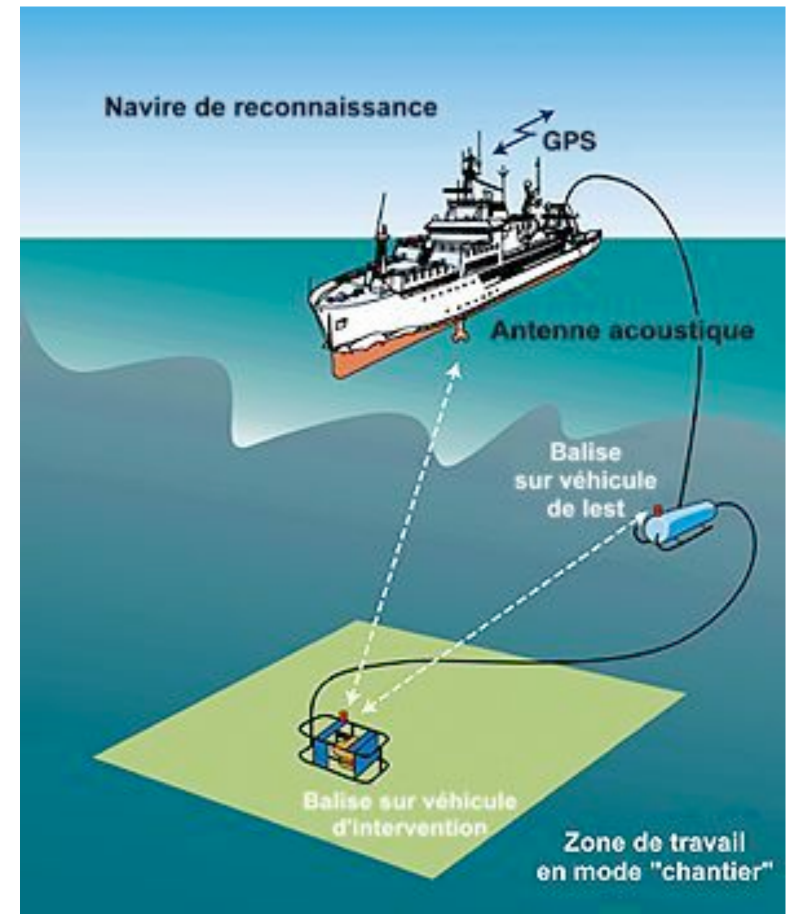




Trilateration (Wikipedia)

# Cooperative Navigation: single range navigation

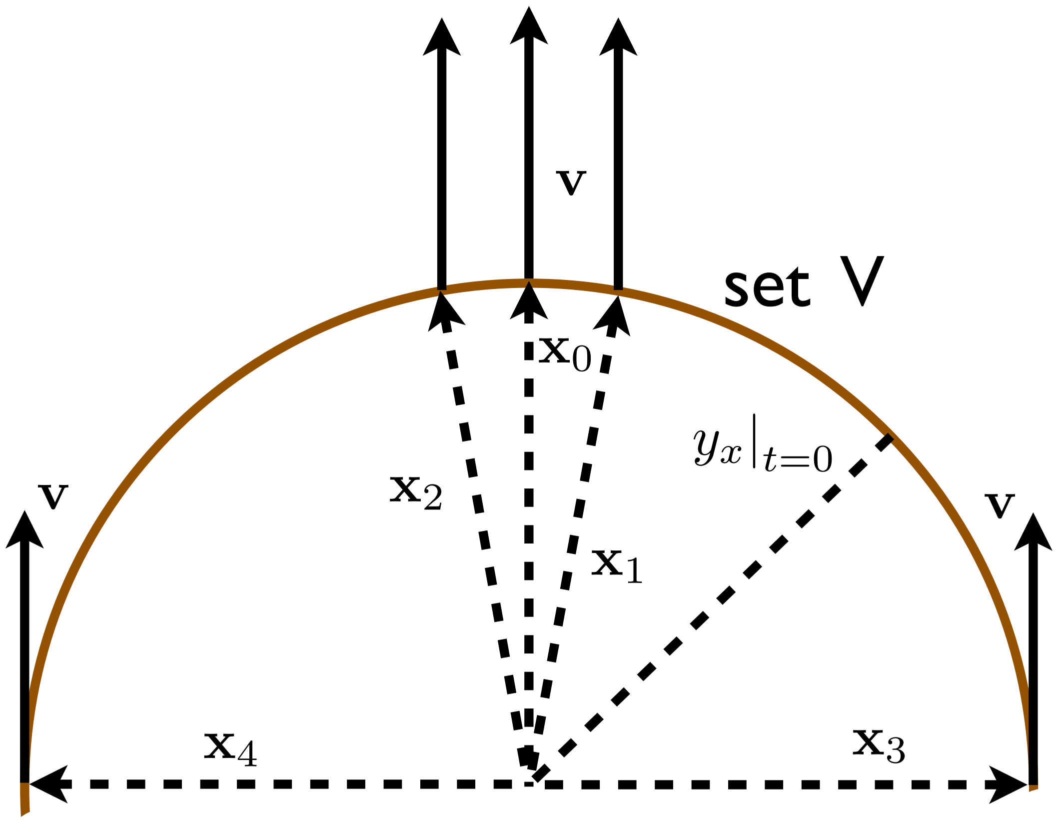


LBL (Kongsberg web site)




USBL (IFREMER web site)

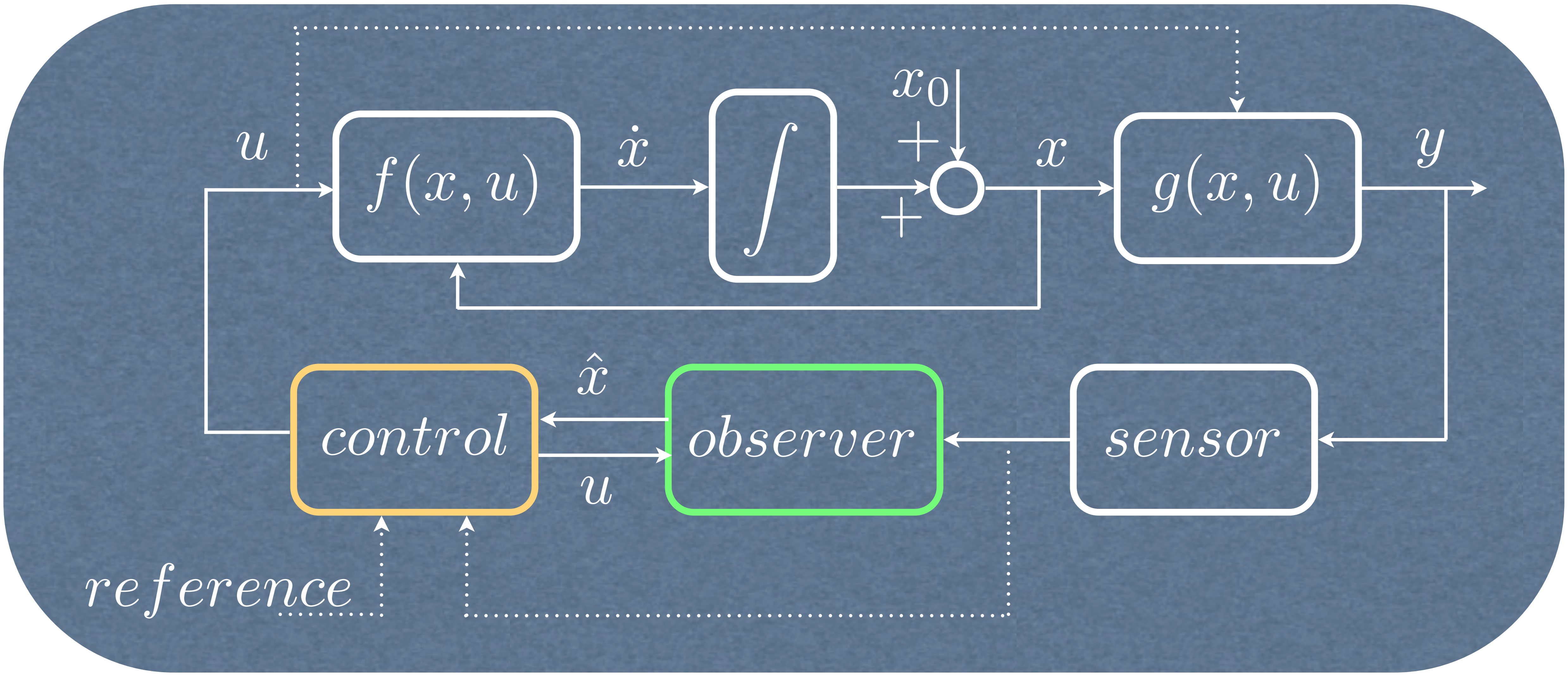
# Single range navigation



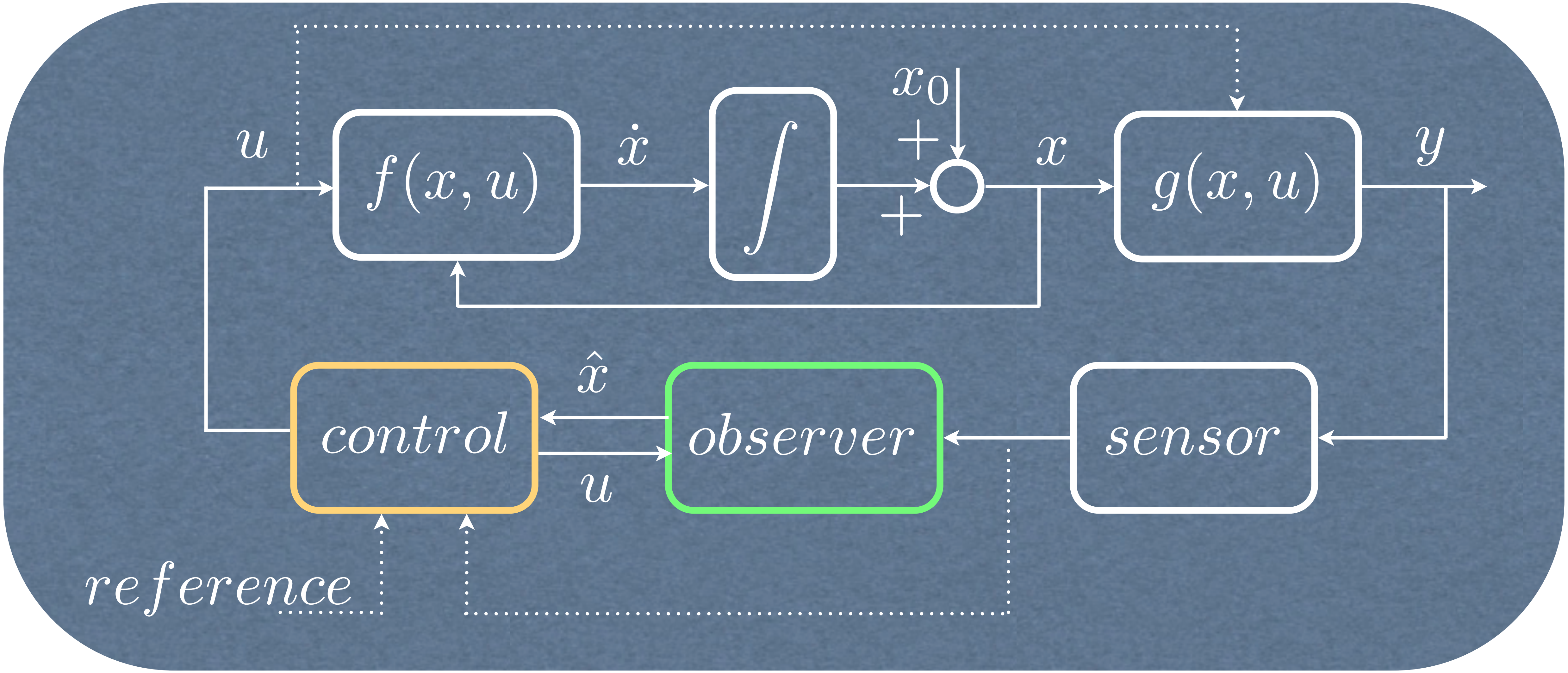

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{v} \\ y &= k\,\|\mathbf{x}\|^{\alpha} : \\ & \quad k > 0, \alpha = \{1, 2\}\end{aligned}$$

What velocity profile guarantees observability for a given initial position?

# Observers & Observability



# Observers & Observability




Non-observability == existence of indistinguishable states, i.e. different initial states that generate the same output for a given input.

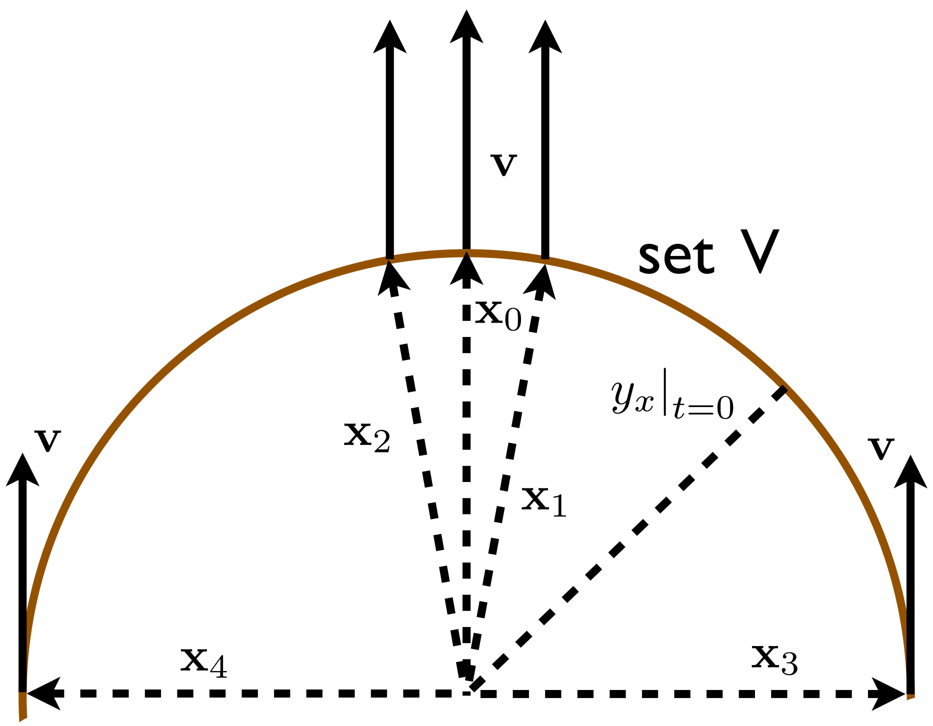
# LTI Case Observability

- Does not depend on the input, i.e. it relates to the free response only;
- Is a global property;
- Does not depend on time.

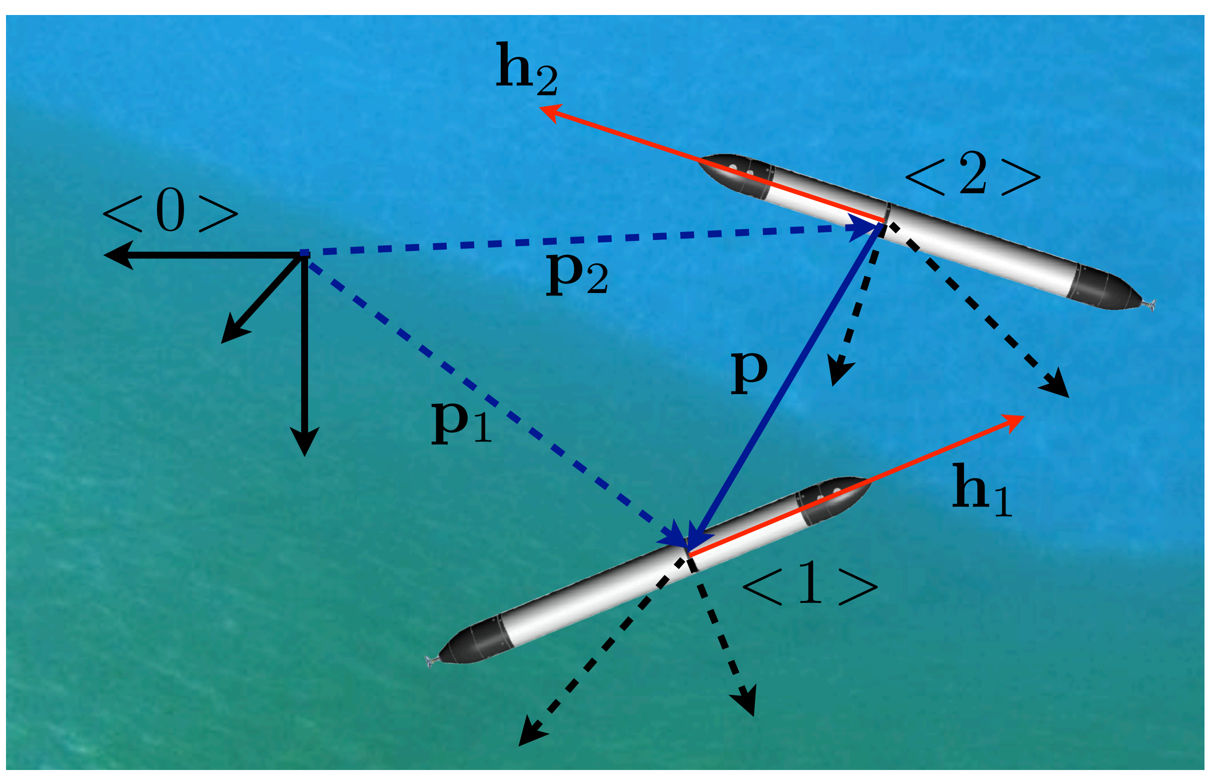
# Nonlinear Case

- Generally depends on the input;
- Generally it is a local property;
- Generally depends on time.

# Studied cases




$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{v} \\
y &= k\,\|\mathbf{x}\|^{\alpha} : \\
& \quad k > 0, \alpha = \{1, 2\}
\end{aligned}
$$




$$\|\mathbf{h}_i(t)\| = 1$$

$$\dot{\mathbf{p}}_i = u_i(t)\,\mathbf{h}_i(t)$$

$$\dot{\mathbf{h}}_i = \boldsymbol{\omega}_{i/0}(t) \times \mathbf{h}_i(t)$$

$$y = \frac{1}{2}\,\|\dot{\mathbf{p}}_i - \dot{\mathbf{p}}_j\|^2$$

# Methods (in a nutshell)



$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{v} \\ y &= k\,\|\mathbf{x}\|^{\alpha} : \\ & k > 0, \alpha = \{1,2\} \end{aligned}$$

$$\|\mathbf{h}_i(t)\| = 1$$
$$\dot{\mathbf{p}}_i = u_i(t)\,\mathbf{h}_i(t)$$
$$\dot{\mathbf{h}}_i = \boldsymbol{\omega}_{i/0}(t) \times \mathbf{h}_i(t)$$
$$y = \frac{1}{2}\,\|\dot{\mathbf{p}}_i - \dot{\mathbf{p}}_j\|^2$$

Consider a higher dimensional Linear Time Varying (LTV) realization.

Apply standard tools (Gramian observability matrix).

# Methods (i... a nutshell)

State Augmentation

$$\dot{\mathbf{x}} = \mathbf{v}$$

$$y = k\,\|\mathbf{x}\|^{\ldots}$$

$$k > \ldots \in \{1, 2\}$$

$$\|\mathbf{h}_i(t\ldots$$

$$\dot{\mathbf{p}} \ldots)\,\mathbf{h}_i(t)$$

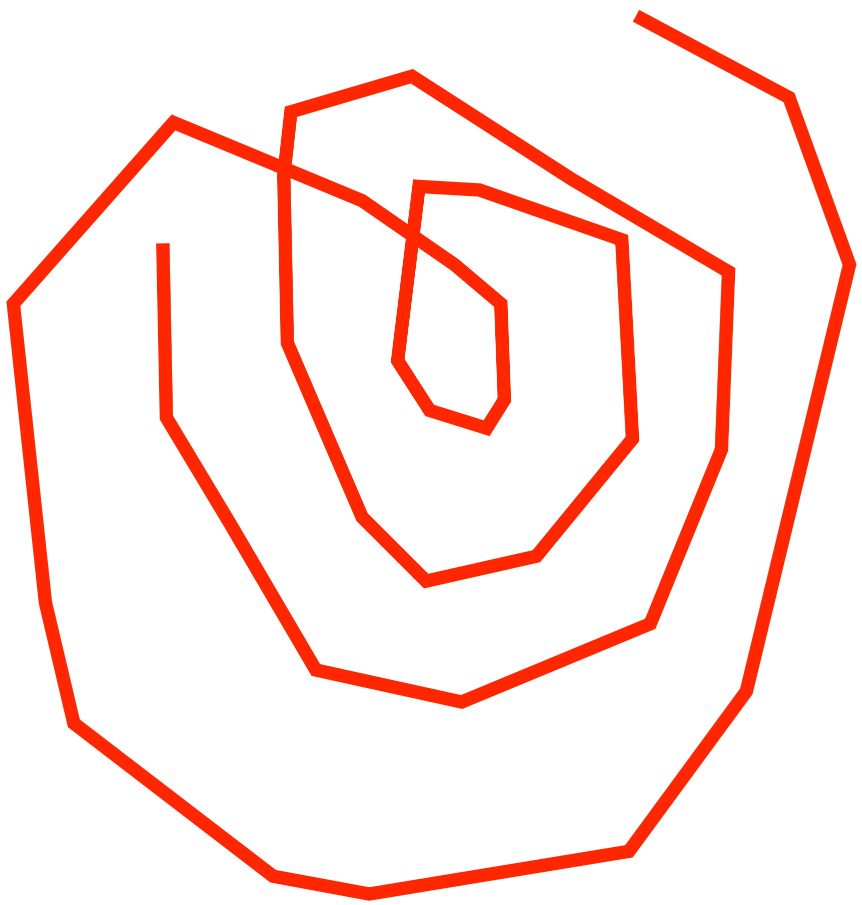
$$\ldots\omega_{i/0}(t) \times \mathbf{h}_i(t)$$

$$\ldots = \frac{1}{2}\,\|\dot{\mathbf{p}}_i - \dot{\mathbf{p}}_j\|^2$$

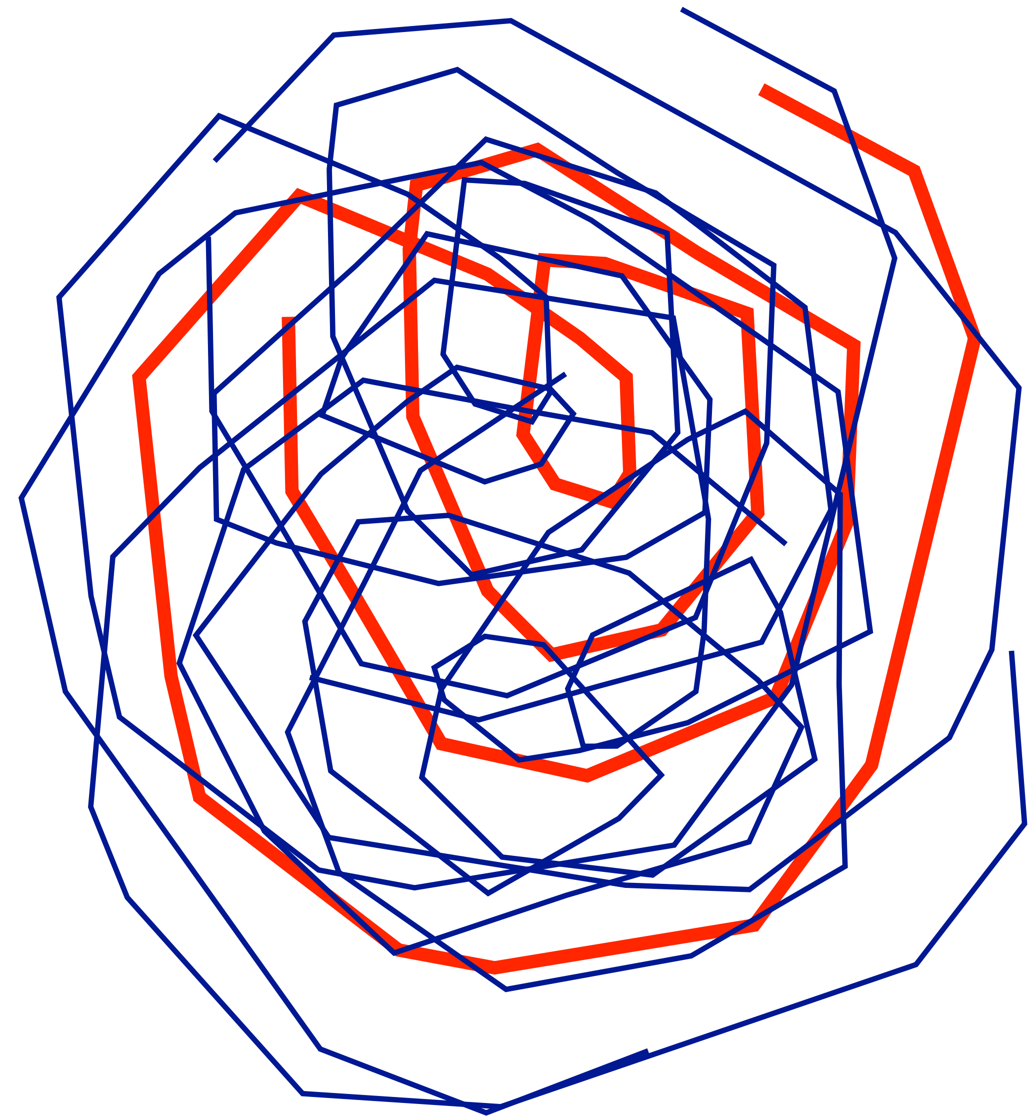
Generate a higher dimensional Linear Time Varying (LTV) realization.

Apply standard tools (Gramian observability matrix).

# Methods (in a nutshell)

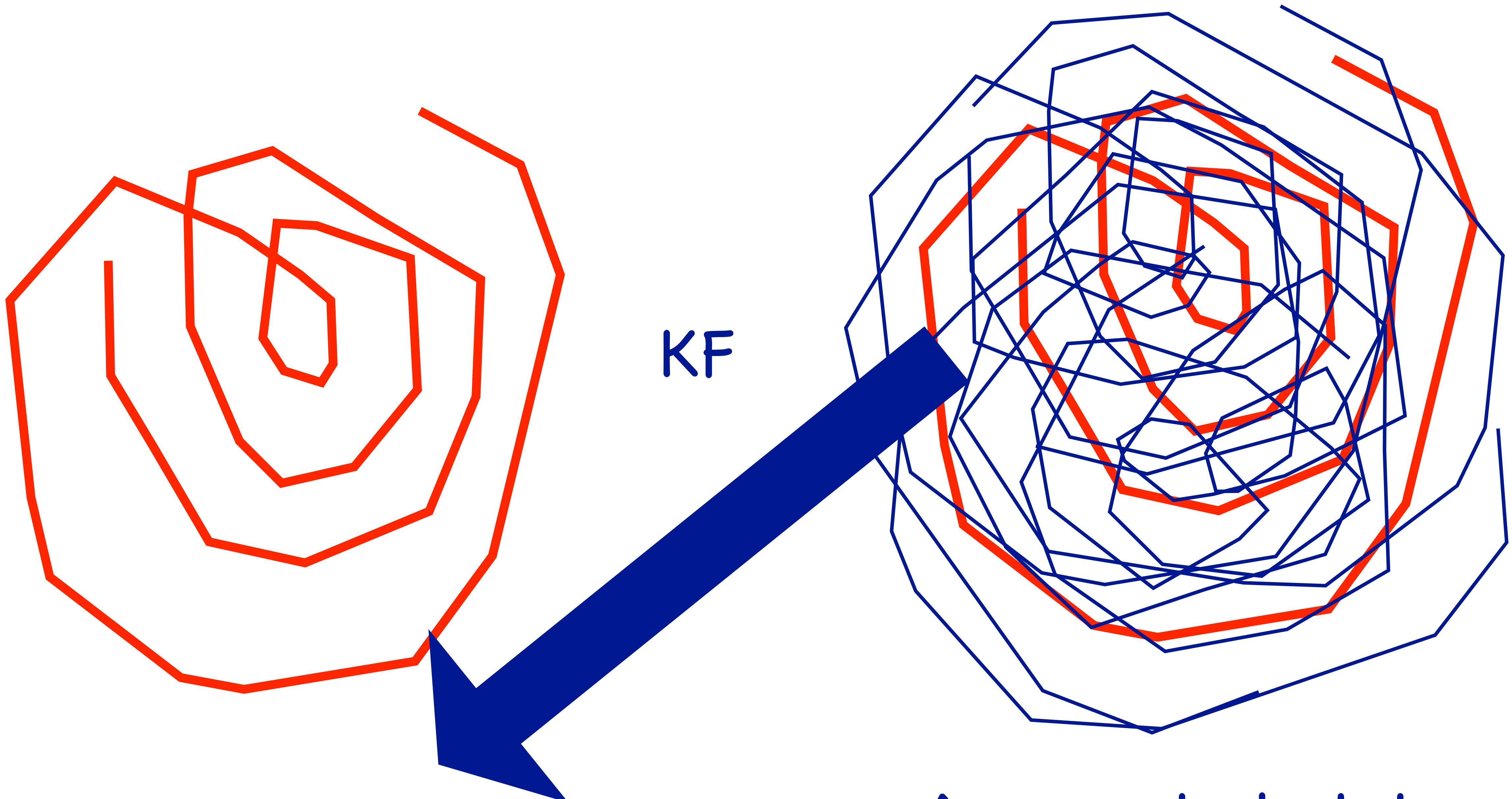


Original Nonlinear System



Augmented state LTV System

# Methods (in a nutshell)




Original Nonlinear System

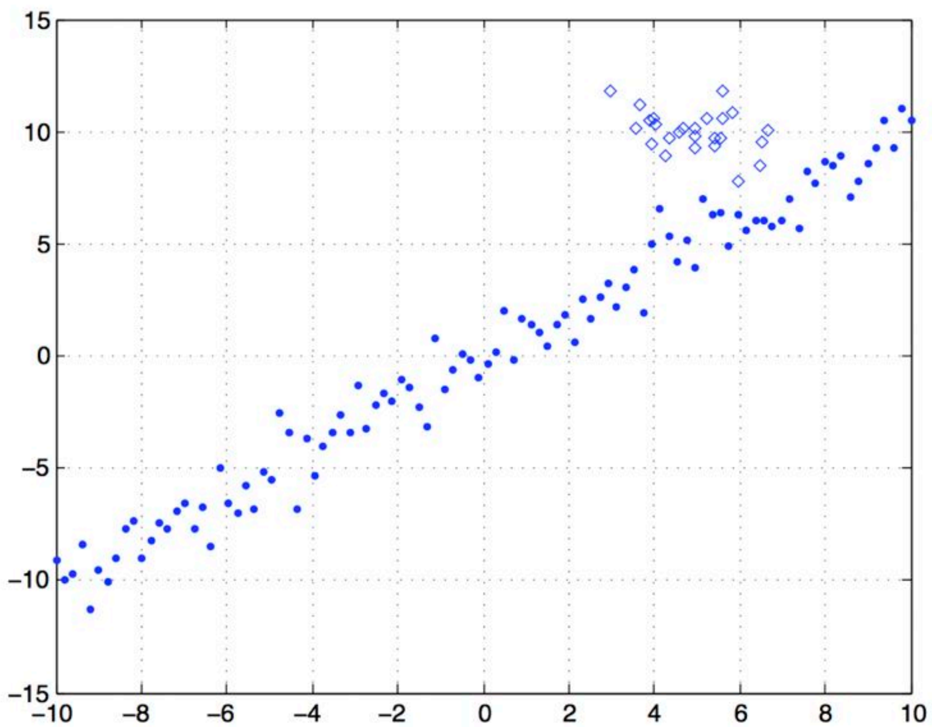
Augmented state LTV System

# An additional issue

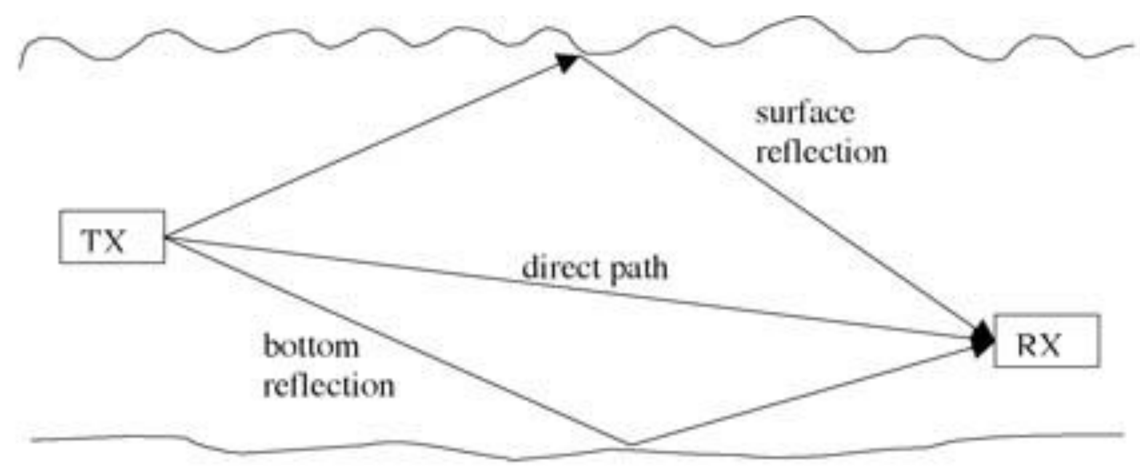
http://en.wikipedia.org

# Measurement Outliers!!

# An additional issue




http://www.ieeeoes.org/pubs/newsletters/oes/html/spring06/underwater.html

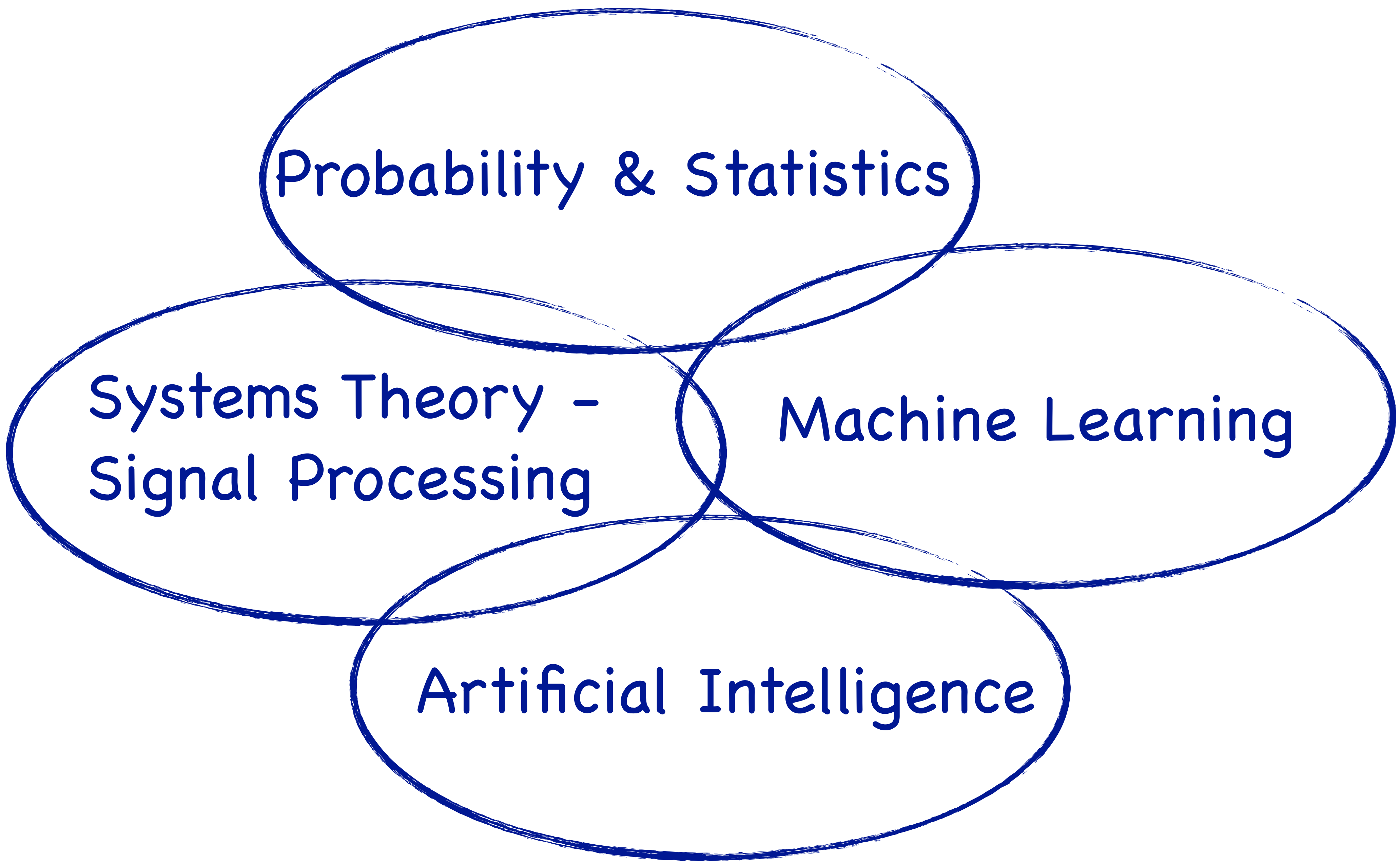



# Outliers!!

# Outlier Robust State Estimation

Mount Everest (Wikipedia)

# High Level Classification



# High Level Classification

Cost minimization (robust implementation of Least Squares / Kalman / Luenberger type filters, Winsorization / Huberization )

Voting Schemas (Hough transform, Least Median of Squares)

# Least Median of Squares



Peter J. Rousseeuw, Least Median of Squares Regression, Journal of the American Statistical Association December 1984, Volume 79, Number 388 Theory and Methods Section

$$\underset{\hat{\theta}}{\text{minimize}} \sum_{i=1}^{n} r_i^2,$$

$$\underset{\hat{\theta}}{\text{minimize}} \ \underset{i}{\text{med}} \ r_i^2.$$

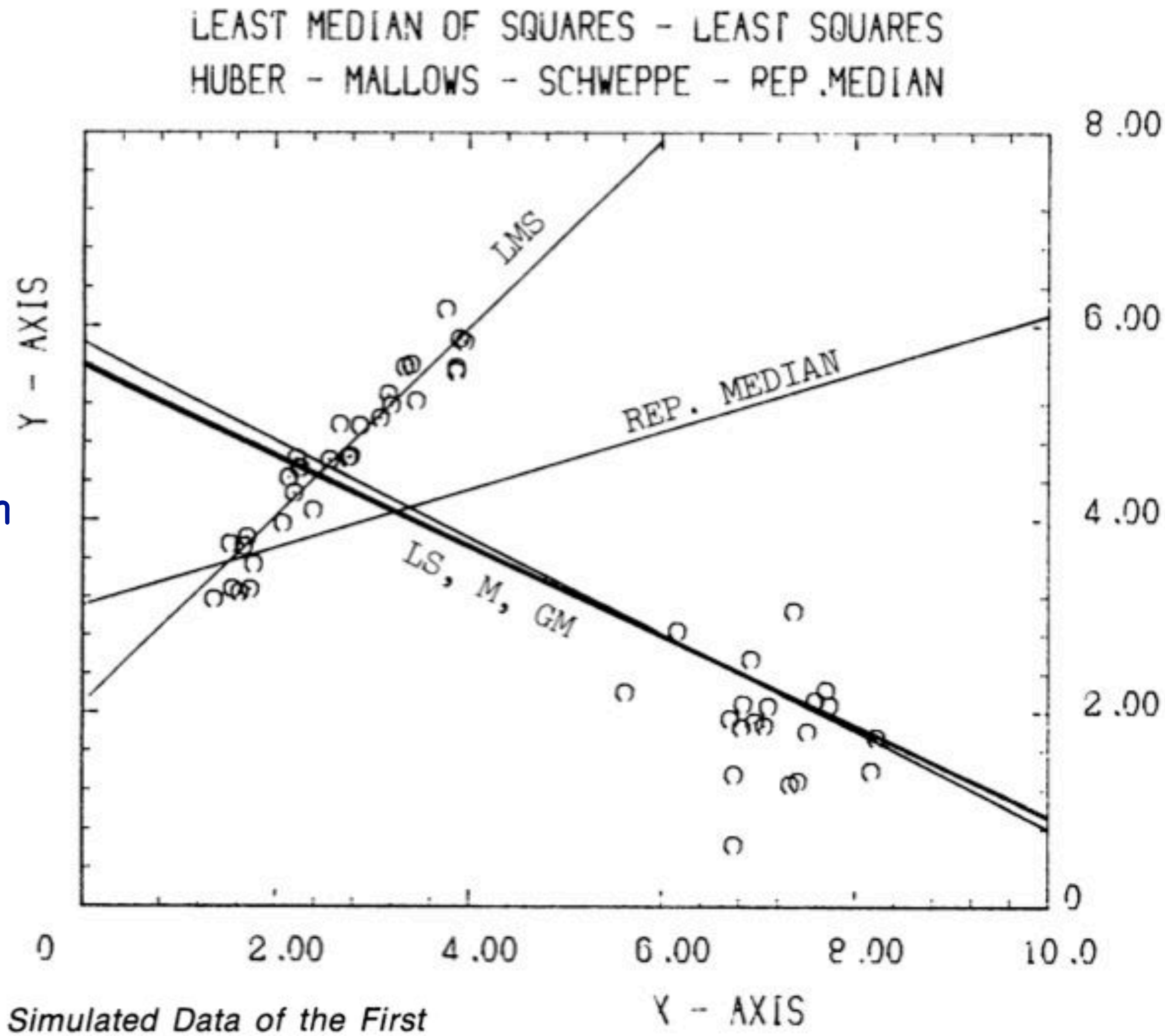
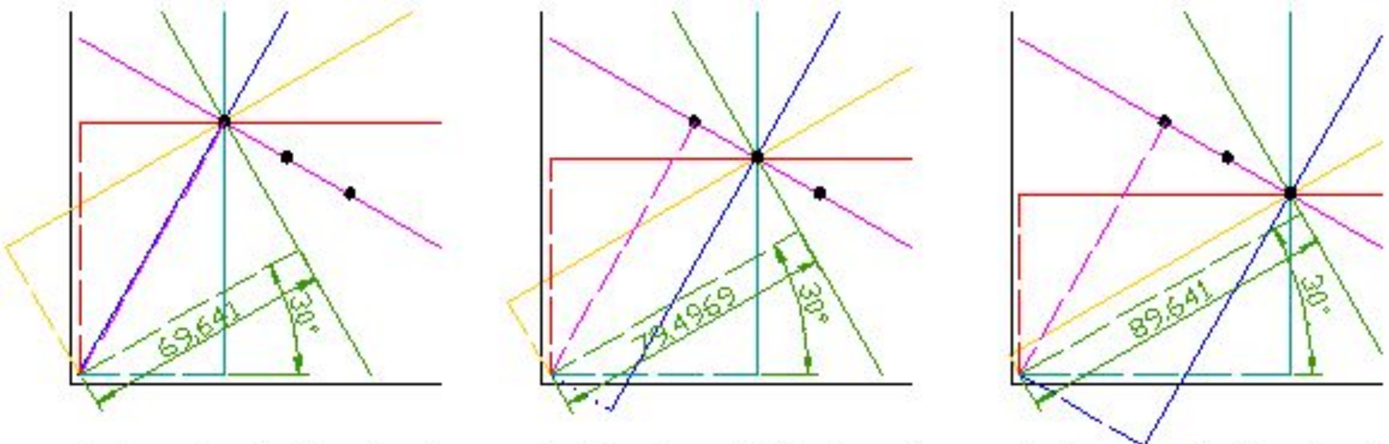



Figure 2. Regression Lines for the Simulated Data of the First Example, Using Six Methods. (LMS = least median of squares; LS = least squares; M = Huber's M estimator; GM = Mallows's and Schweppe's G-M estimator; REP. MEDIAN = repeated median; ⊙ = 30 "good" points generated according to a linear relation $y_i = x_i + 2 + e_i$ and 20 "bad" points in a spherical cluster around (7, 2).

# Hough Transform



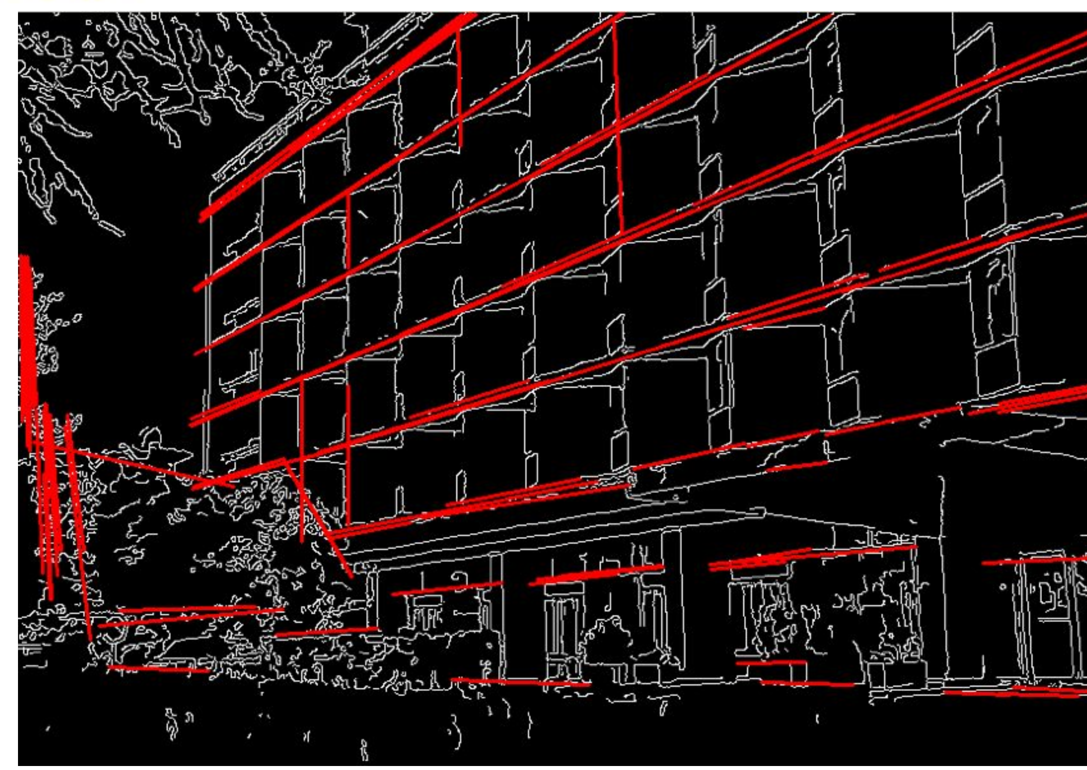

| Angle | Dist. |
|---|---|
| 0 | 40 |
| 30 | 69.6 |
| 60 | 81.2 |
| 90 | 70 |
| 120 | 40.6 |
| 150 | 0.4 |

| Angle | Dist. |
|---|---|
| 0 | 57.1 |
| 30 | 79.5 |
| 60 | 80.5 |
| 90 | 60 |
| 120 | 23.4 |
| 150 | −19.5 |

| Angle | Dist. |
|---|---|
| 0 | 74.6 |
| 30 | 89.6 |
| 60 | 80.6 |
| 90 | 50 |
| 120 | 6.0 |
| 150 | −39.6 |

http://en.wikipedia.org

http://docs.opencv.org/modules/imgproc/doc/feature_detection.html



# Kalman – Like

J. R. Statist. Soc. B (1984), 46, No. 2, pp. 149–192

**Iteratively Reweighted Least Squares for Maximum Likelihood Estimation, and some Robust and Resistant Alternatives**

By P. J. GREEN

Stat Papers (2014) 55:93–123
DOI 10.1007/s00362-012-0496-4

REGULAR ARTICLE

**Robust Kalman tracking and smoothing with propagating and non-propagating outliers**

Peter Ruckdeschel · Bernhard Spangl · Daria Pupashenko

2011 IEEE International Conference on Robotics and Automation
Shanghai International Conference Center
May 9-13, 2011, Shanghai, China

**An Outlier-Robust Kalman Filter**

Gabriel Agamennoni, Juan I. Nieto and Eduardo M. Nebot

Automatica, Vol. 16, pp. 53–63
Pergamon Press Ltd. 1980. Printed in Great Britain
International Federation of Automatic Control

**Robust Identification***

B. T. POLJAK†, and JA. Z. TSYPKIN†

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 44, NO. 6, JUNE 1999

**Robust Estimation with Unknown Noise Statistics**

Željko M. Đurović and Branko D. Kovačević

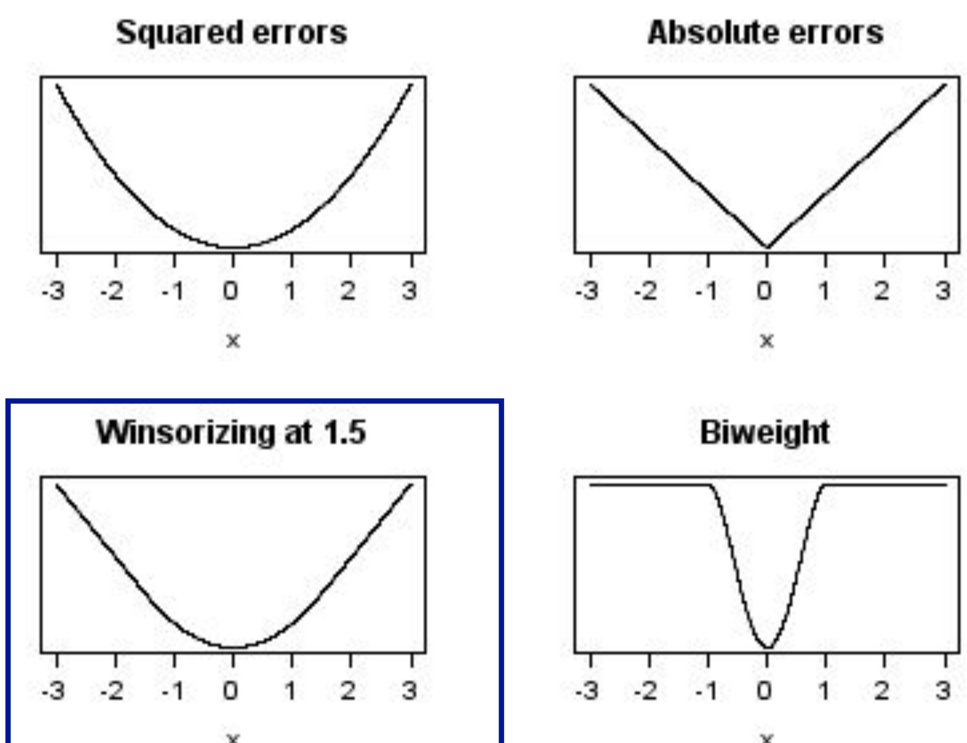
...d de Estadística e Investigación Operativa
...997) Vol. 6, No. 2, pp. 379–395

**Kalman filter with outliers and missing observations**

**T. Cipra**
*Department of Statistics, Charles University of Prague*
*Sokolovska 83; 186 00 Prague 8, Czech Republic*
**R. Romera**
*Departamento de Estadística y Econometría*
*Universidad Carlos III de Madrid*
*Madrid 126, 28903 Getafe, Spain.*

# Kalman – Like

http://en.wikipedia.org



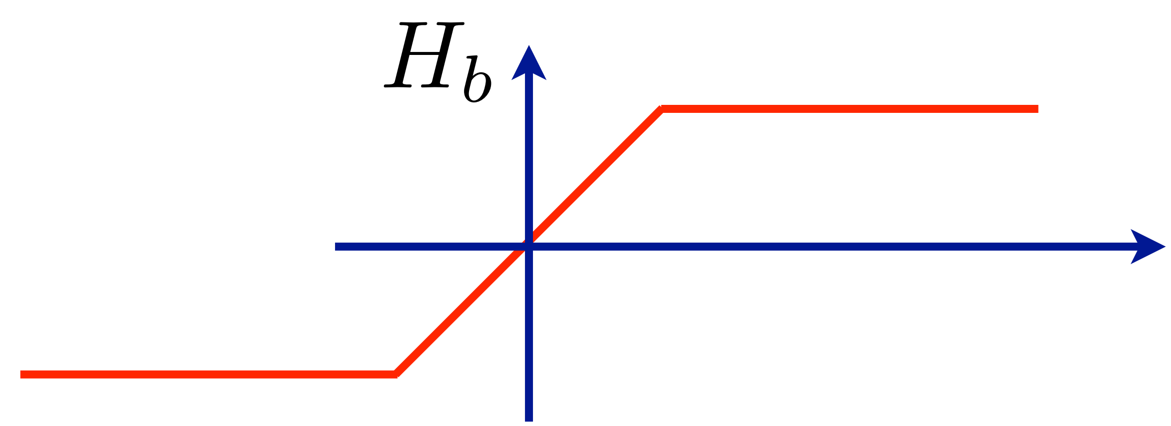

Huber Loss function (M-estimators)

Stat Papers (2014) 55:93–123
DOI 10.1007/s00362-012-0496-4
REGULAR ARTICLE

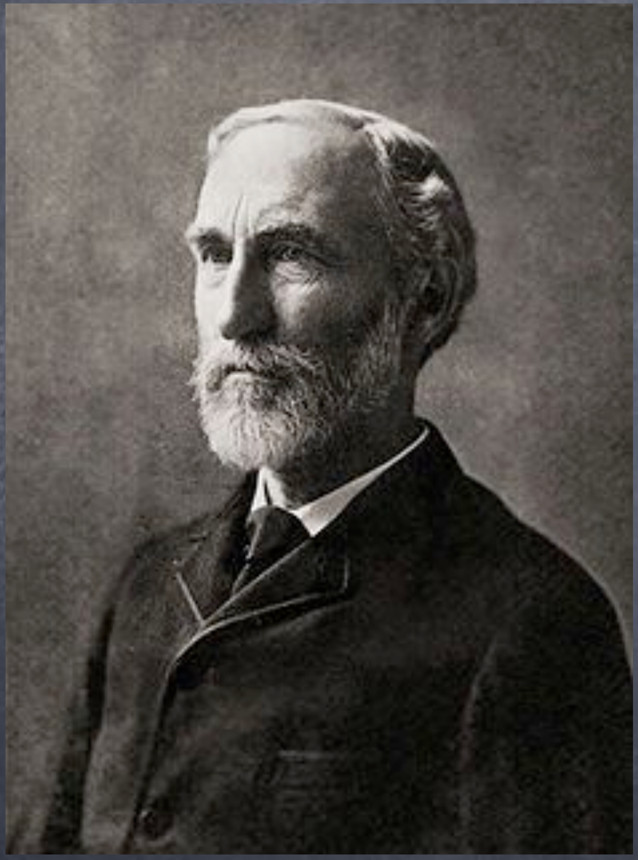
**Robust Kalman tracking and smoothing with propagating and non-propagating outliers**

**Peter Ruckdeschel · Bernhard Spangl · Daria Pupashenko**

$$X_{t|t} = X_{t|t-1} + H_b(K_t \Delta Y_t).$$



# Proposed approach (Gibbs) Entropy



Josiah Willard Gibbs (1839 - 1903)

$$p_i \in [0, 1]$$

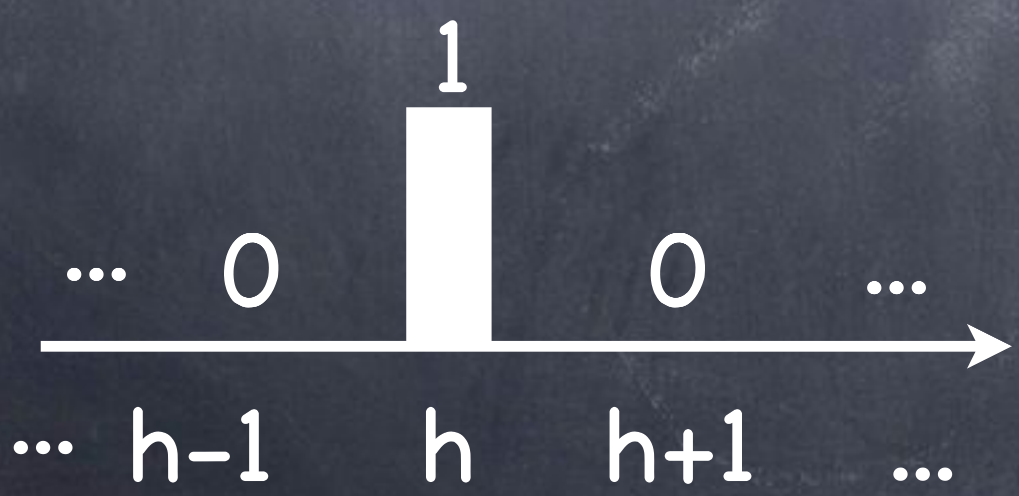
$$\sum_i p_i = 1$$

$$S = -k \sum_i p_i \ln p_i$$

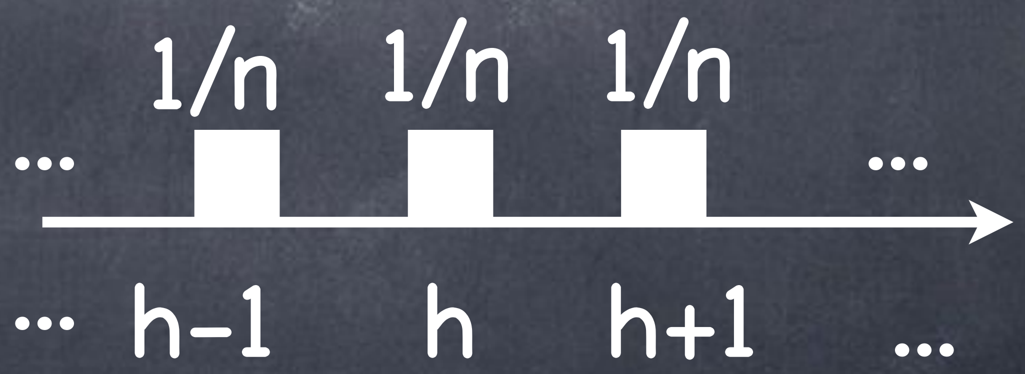
# Proposed approach (Gibbs) Entropy

$$\sum_i p_i = 1 \qquad S = -k \sum_i p_i \ln p_i$$

Min S = 0




Max S = k log(n)



# Proposed approach (1999, Bonn)


$$r_i = y_i - \hat{y}_i(\boldsymbol{\theta})$$

$$\hat{\boldsymbol{\theta}}_{LS} = \arg\min_{\boldsymbol{\theta}} \frac{1}{N} \sum_{i=1}^{N} r_i^2$$

# Proposed approach (1999, Bonn)


$$r_i = y_i - \hat{y}_i(\boldsymbol{\theta})$$

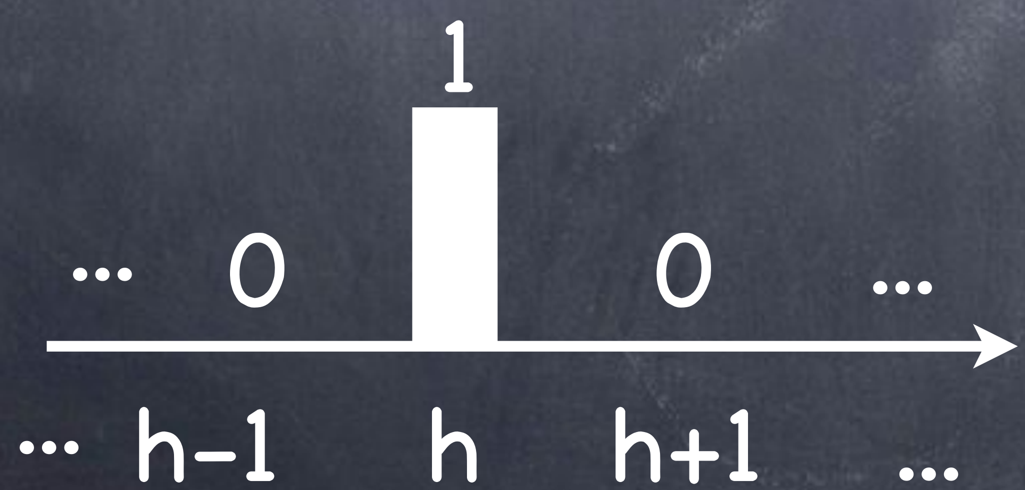
$$\hat{\boldsymbol{\theta}}_{LS} = \arg\min_{\boldsymbol{\theta}} \frac{1}{N} \sum_{i=1}^{N} r_i^2$$

$$H = -\frac{1}{\log N} \sum_i^N \frac{r_i^2}{\sum_j^N r_j^2} \log \frac{r_i^2}{\sum_j^N r_j^2}$$

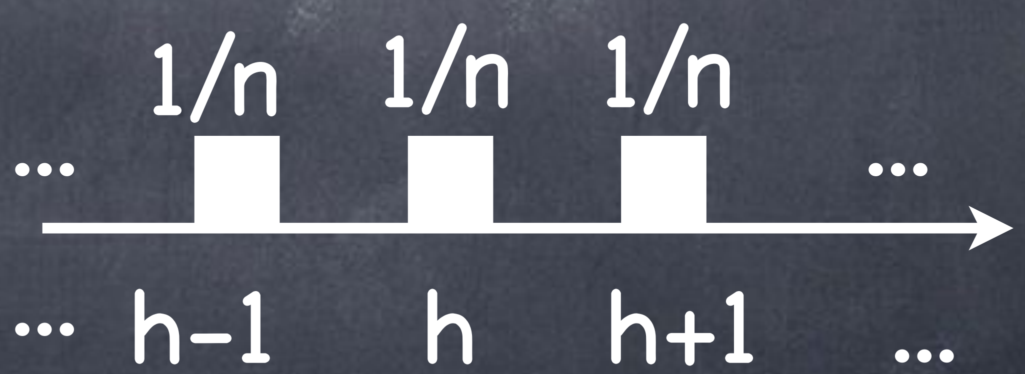
# Proposed approach (Gibbs) Entropy

$$\sum_i p_i = 1 \qquad S = -k \sum_i p_i \ln p_i$$

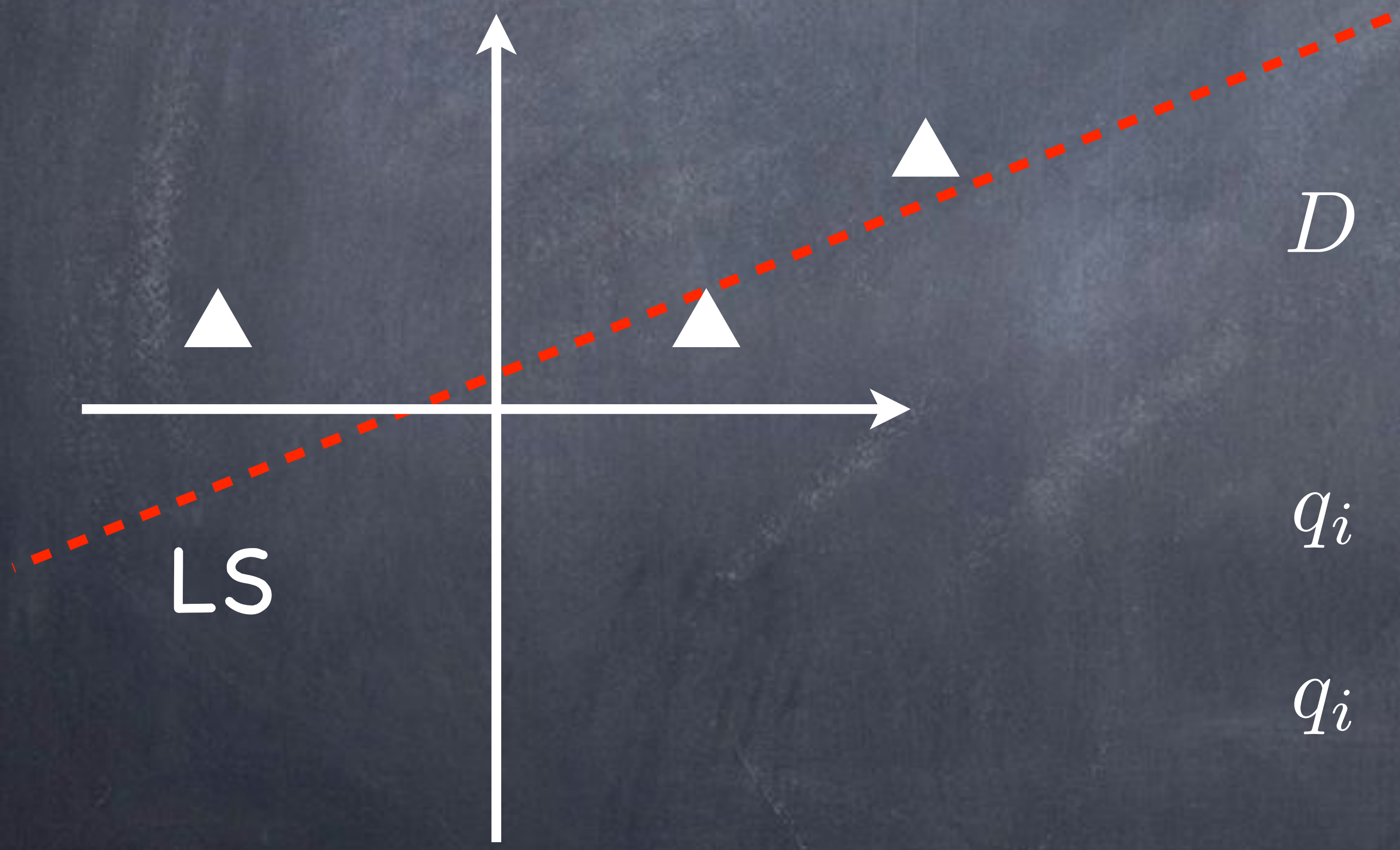
Min S = 0




Max S = k log(n)



# Proposed approach (Gibbs) Entropy



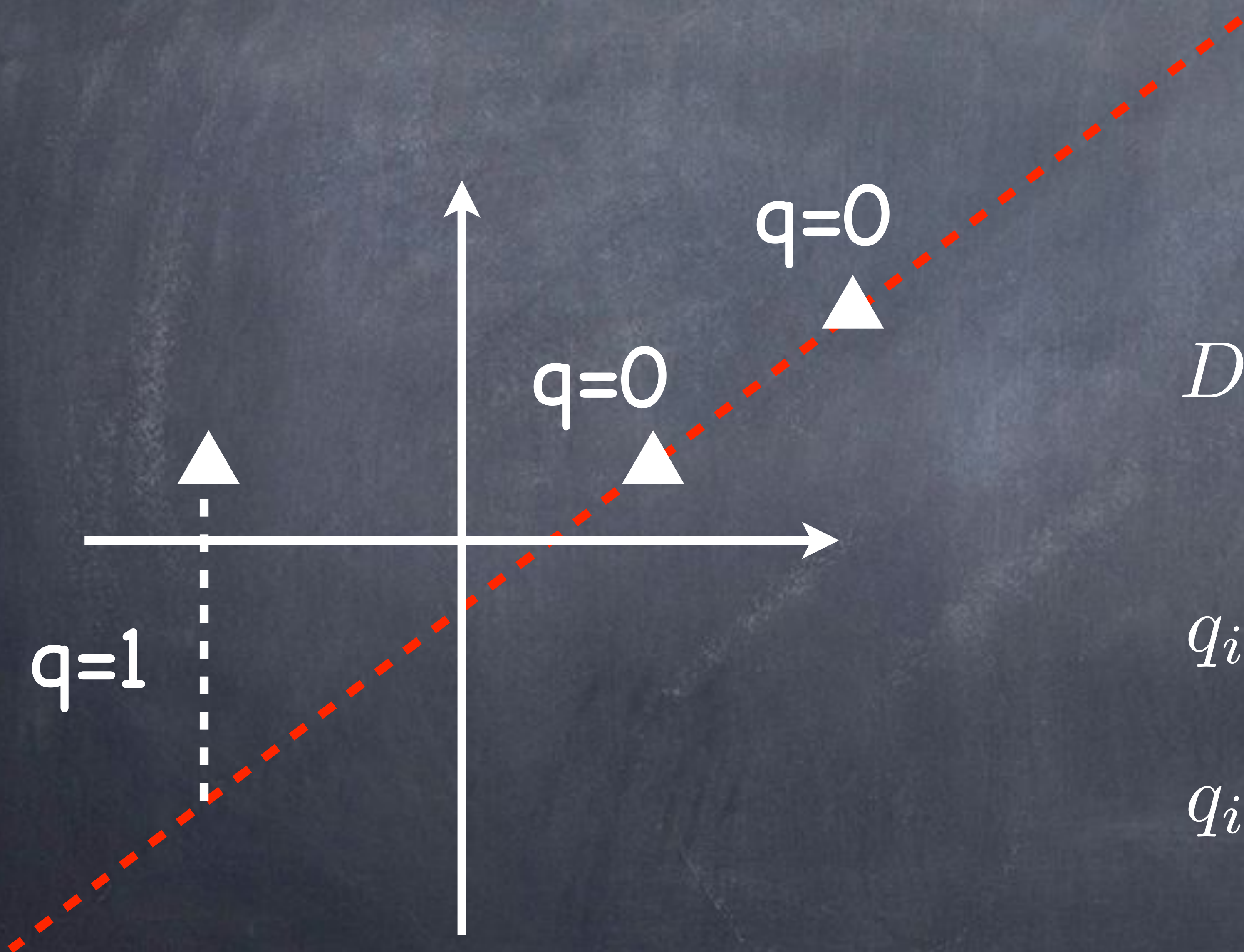

$$D = \sum_{h=1}^{n} r_h^2$$

$$q_i = \frac{r_i^2}{D}$$

$$q_i \in [0,1] \text{ and } \sum_i q_i = 1$$

Integrated Systems for Marine Environment

# Proposed approach (Gibbs) Entropy

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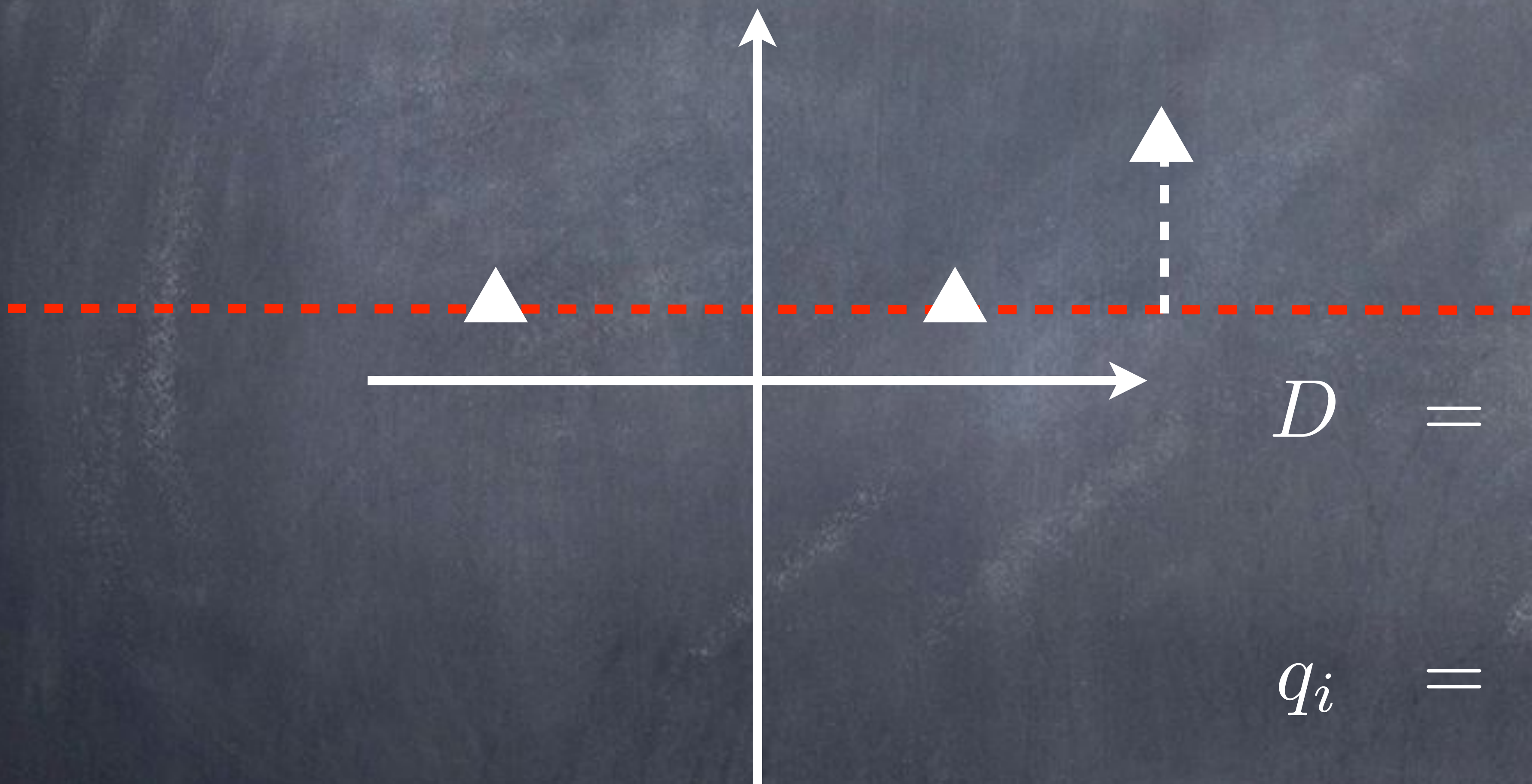

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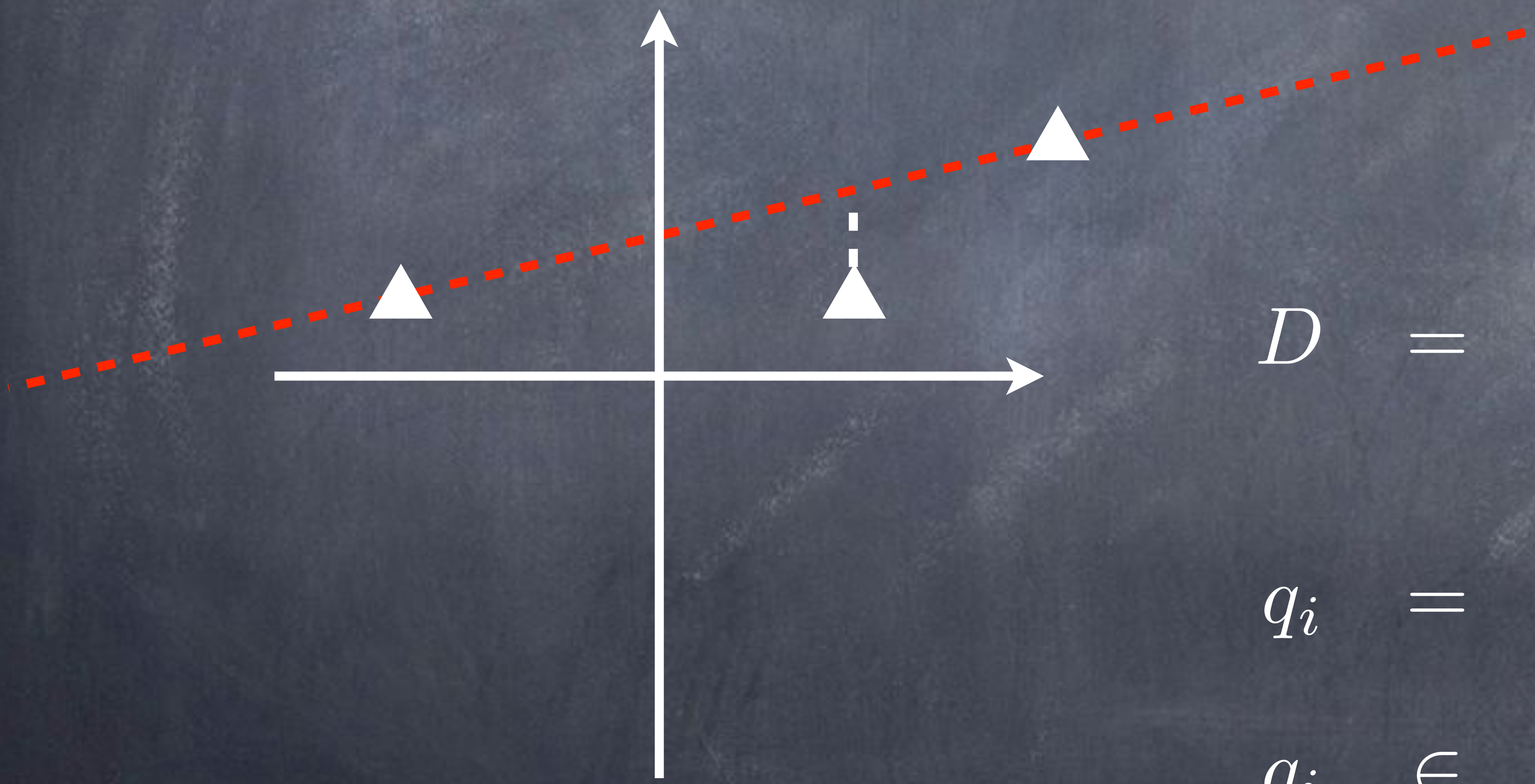
# Proposed approach (Gibbs) Entropy



$$D = \sum_{h=1}^{n} r_h^2$$

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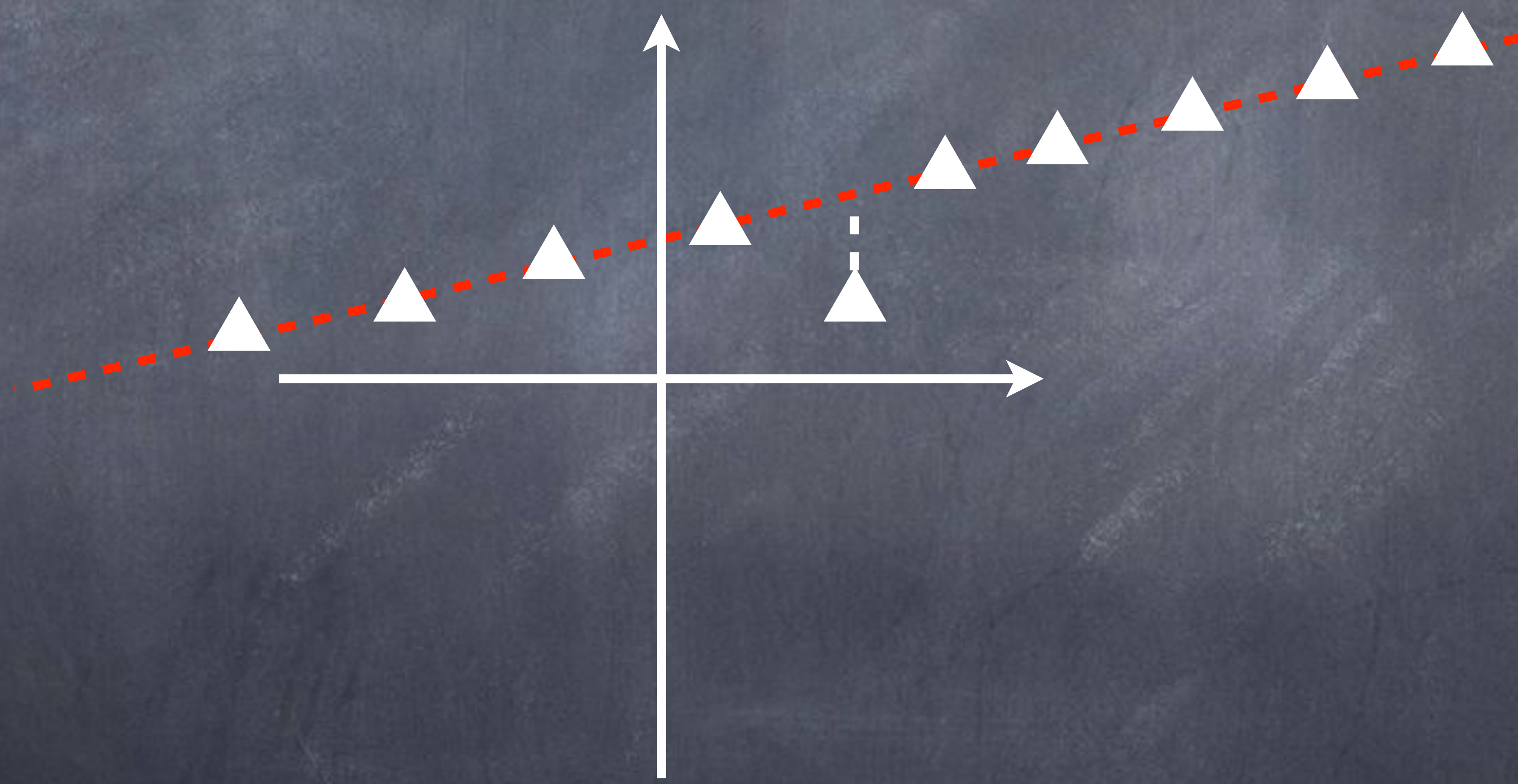
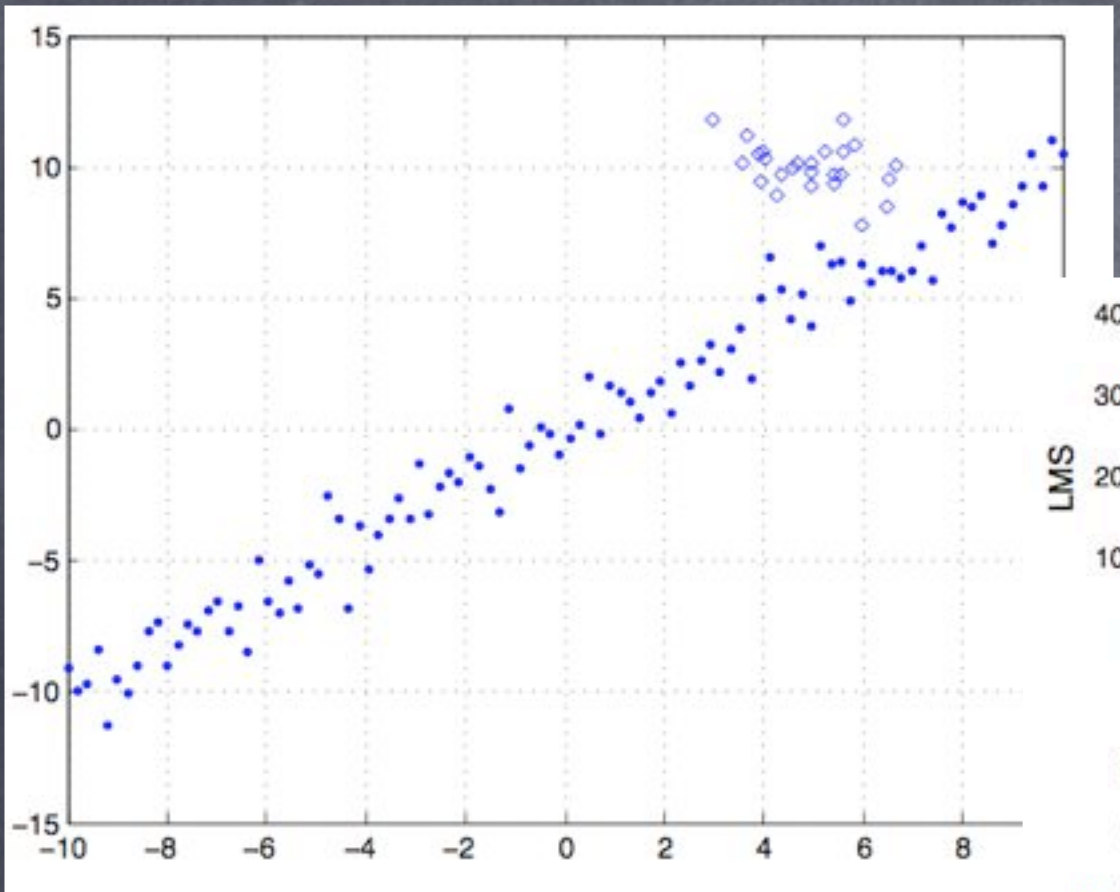
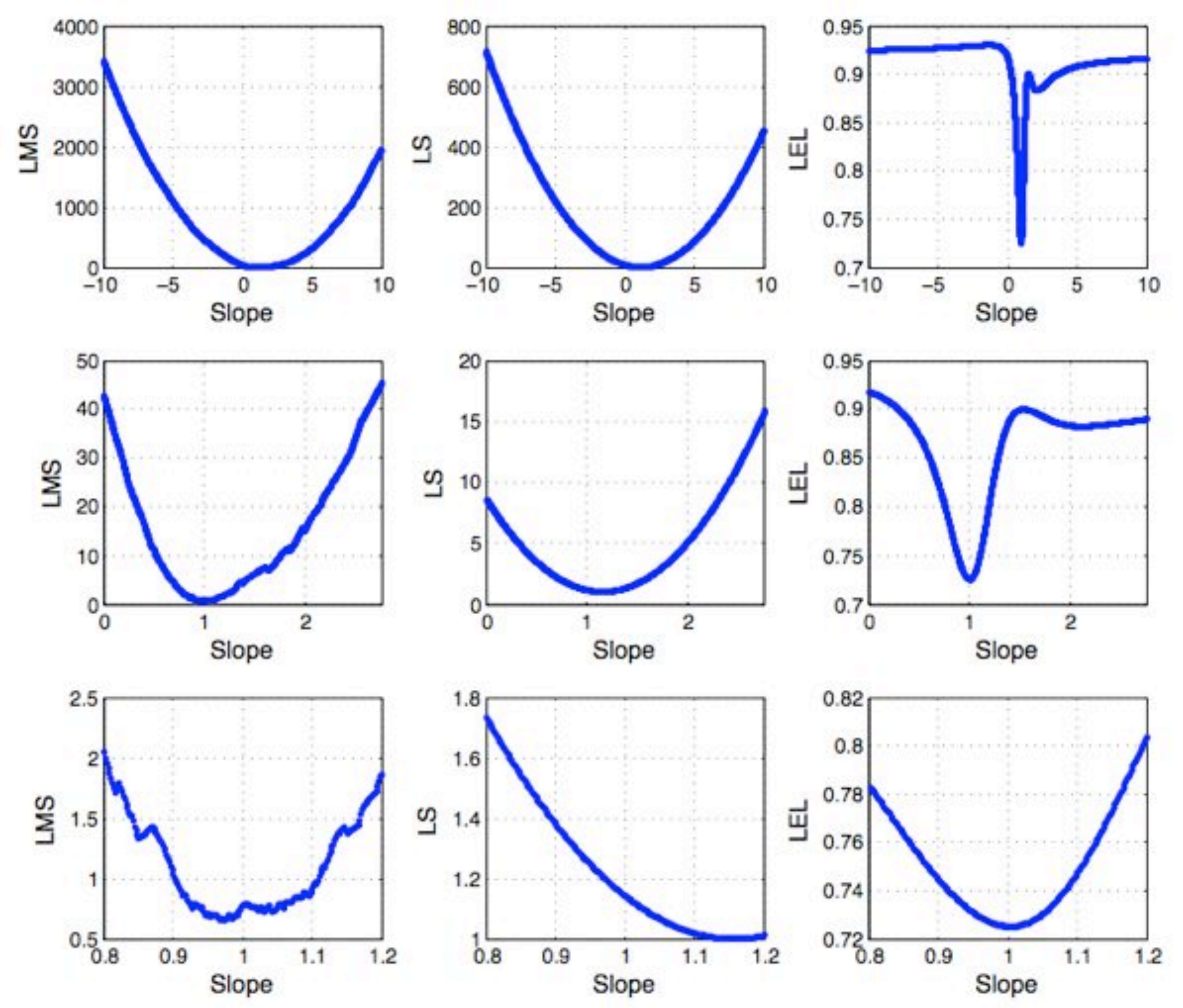
$$q_i \in [0,1] \text{ and } \sum_i q_i = 1$$

# Proposed approach
# (Gibbs) Entropy








$$y = \theta x$$

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*Article*

# An Entropy-Like Estimator for Robust Parameter Identification

**Giovanni Indiveri**

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2011 18th IEEE International Conference on Image Processing

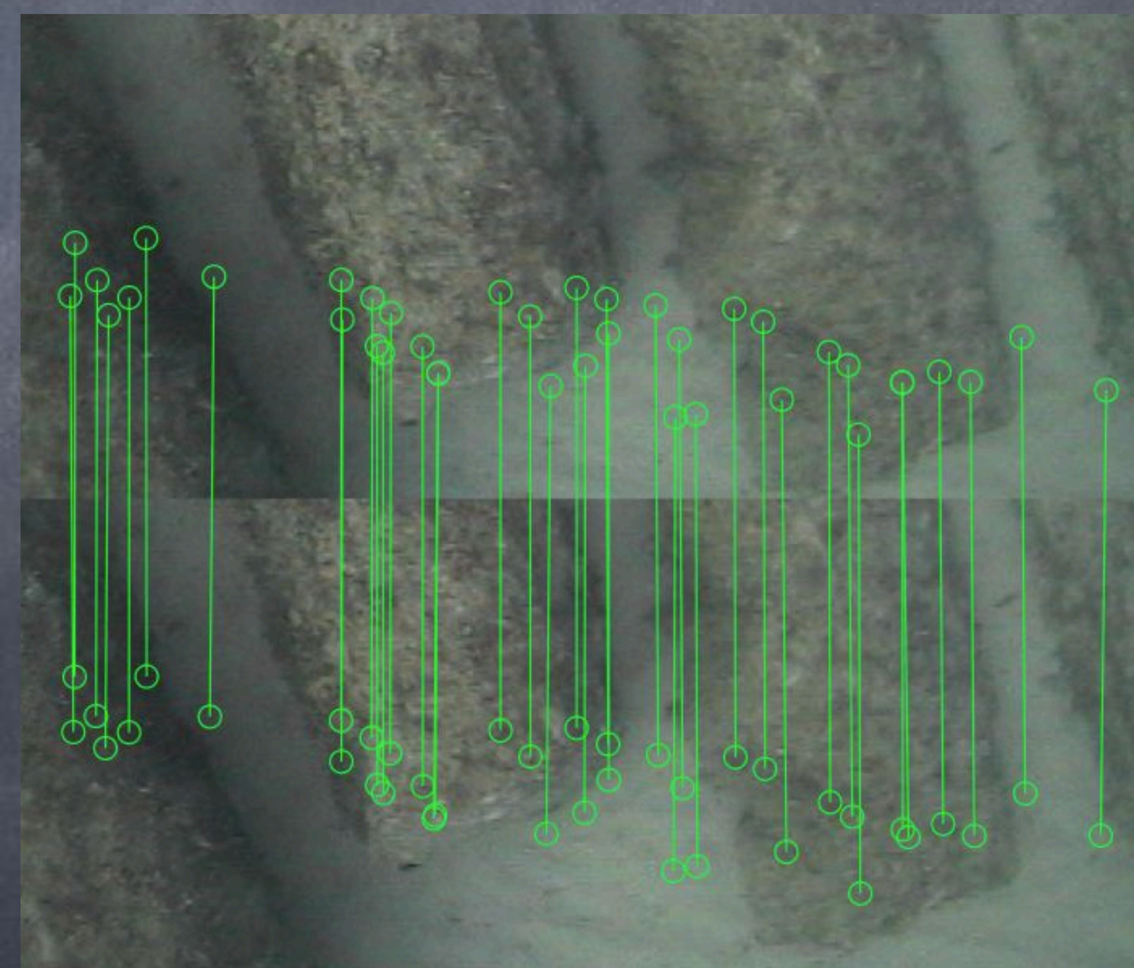
**RANSAC-LEL: AN OPTIMIZED VERSION WITH LEAST ENTROPY LIKE ESTIMATORS**

*Cosimo Distante*

Istituto Nazionale di Ottica - CNR
Arnesano (Le), Italy

*Giovanni Indiveri*

Dipartimento Ingegneria Innovazione
Università del Salento via Monteroni, 73100 Lecce, Italy



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Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/acs

# Output Outlier Robust State Estimation

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# Kalman – LEL

Dynamic Linear Model $\begin{cases} \mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k = C_k \mathbf{x}_k + \mathbf{v}_k \end{cases}$

$\mathbf{w}_k \sim N(0, \boldsymbol{Q})$
$\mathbf{v}_k \sim N(0, \boldsymbol{R})$

## Kalman Filter

$$\hat{\mathbf{x}}_{k+1|k+1} = \arg\min_{\mathbf{x}_{k+1}} J_{k+1}$$

$$J_{k+1} = \underbrace{\frac{1}{2}(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k})^T P^{-1}_{k+1|k}(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k})}_{J_{dynamical\ model}} + \underbrace{\frac{1}{2}(\mathbf{r}^{\top}_{k+1} R^{-1} \mathbf{r}_{k+1})}_{J_{observations\ model}}$$

$$\mathbf{r}_{k+1} = \mathbf{y}_{k+1} - C_{k+1}\mathbf{x}_{k+1}$$

## Kalman-LEL Filter

$$\hat{\mathbf{x}}_{k+1|k+1} = \arg\min_{\mathbf{x}_{k+1}} J_{k+1}$$

$$J_{k+1} = \underbrace{\frac{1}{2}\left((\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k})^T P^{-1}_{k+1|k}(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k})\right)}_{J_{dynamical\ model}} + \underbrace{\alpha H_{k+1}(\mathbf{r}_1, \ldots, \mathbf{r}_{k+1})}_{J_{LEL}}$$

$$\mathbf{r}_i = \mathbf{y}_i - C_i \hat{\mathbf{x}}_i; \quad i = 1, \ldots, k+1$$

# Kalman - LEL

**LEL Functional Cost Approximation**

$$H_{k+1}(\mathbf{x}) = H_{k+1}(\hat{\mathbf{x}}_k) + \nabla_{\mathbf{x}} H_{k+1}(\mathbf{x}) \Big|_{\mathbf{x}=\hat{\mathbf{x}}_k}^{\top} (\mathbf{x} - \hat{\mathbf{x}}_k) + \frac{1}{2}(\mathbf{x} - \hat{\mathbf{x}}_k)^{\top} \mathcal{H}[H_{k+1}(\mathbf{x})] \Big|_{\mathbf{x}=\hat{\mathbf{x}}_k} (\mathbf{x} - \hat{\mathbf{x}}_k) + \mathcal{O}(\|\mathbf{x} - \hat{\mathbf{x}}_k\|^3)$$

$$\nabla_{\mathbf{x}} H_{k+1}(\mathbf{x}) = \frac{2}{D_{k+1}(\mathbf{x}) \log(k+1)} \left( \log \|\mathbf{r}_{k+1}(\mathbf{x})\|^2 - \frac{S_{k+1}(\mathbf{x})}{D_{k+1}(\mathbf{x})} \right) C_{k+1}^T \mathbf{r}_{k+1}(\mathbf{x})$$

$$\mathcal{H}[H_{k+1}(\mathbf{x})] = \frac{2}{D_{k+1}^2(\mathbf{x}) \log(k+1)} \Big[ 2C_{k+1}^T \mathbf{r}_{k+1}(\mathbf{x}) \mathbf{r}_{k+1}^T(\mathbf{x}) C_{k+1} \left( 2 \log \|\mathbf{r}_{k+1}(\mathbf{x})\|^2 - 2\frac{S_{k+1}(\mathbf{x})}{D_{k+1}(\mathbf{x})} - \frac{D_{k+1}(\mathbf{x})}{\|\mathbf{r}_{k+1}(\mathbf{x})\|^2} + 1 \right) + - C_{k+1}^T C_{k+1} \left( D_{k+1}(\mathbf{x}) \log \|\mathbf{r}_{k+1}(\mathbf{x})\|^2 - S_{k+1}(\mathbf{x}) \right) \Big]$$

$$S_{k+1}(\mathbf{x}) = \sum_{j=1}^{k+1} \|\mathbf{r}_j(\mathbf{x})\|^2 \log \|\mathbf{r}_j(\mathbf{x})\|^2$$

# Kalman - LEL

$$\hat{\mathbf{x}}_{k+1} = \arg\min_{\mathbf{x}_{k+1}} \ \tfrac{1}{2}(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k})^\top P_{k+1|k}^{-1}(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k}) + \\ +\alpha\left((H_{k+1}(\hat{\mathbf{x}}_k) + \nabla_{\mathbf{x}_{k+1}} H_{k+1}(\hat{\mathbf{x}}_k)^\top(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_k) + \tfrac{1}{2}(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_k)^\top \mathcal{H}[H_{k+1}(\hat{\mathbf{x}}_k)](\mathbf{x}_{k+1} - \hat{\mathbf{x}}_k)\right)$$

$$\hat{\mathbf{x}}_{k+1|k} = A_k\hat{\mathbf{x}}_k + B_k\mathbf{u}_k \qquad \triangleright \text{ Predictor}$$
$$P_{k+1|k} = A_k P_{k|k} A_k^T + Q$$

$$K_{k+1} = (P_{k+1|k}^{-1} + \alpha\mathcal{H}[H_{k+1}(\hat{\mathbf{x}}_k)])^{-1}\alpha\mathcal{H}[H_{k+1}(\hat{\mathbf{x}}_k)] \qquad \triangleright \text{ Corrector}$$
$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_{k+1|k} + K_{k+1}(\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k+1|k}) + \\ -(P_{k+1|k}^{-1} + \alpha\mathcal{H}[H_{k+1}(\hat{\mathbf{x}}_k)])^{-1}\alpha\nabla H_{k+1}(\hat{\mathbf{x}}_k)$$
$$P_{k+1|k+1} = (P_{k+1|k}^{-1} + \alpha\mathcal{H}[H_{k+1}(\hat{\mathbf{x}}_k)])^{-1}$$

**Implementation Issues**

- Initialization
- Sliding window implementation
- Regularization of LEL functional cost

# Kalman - LEL

$$\mathbf{x}_k = A_{k-1}\mathbf{x}_{k-1} + B_{k-1}\mathbf{u}_{k-1} + \mathbf{w}_{k-1}$$

$$\mathbf{y}_k = C_k\mathbf{x}_k + \mathbf{v}_k$$

State equation
zero mean
gaussian noise
covariance

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad Q = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

$$(R_k)_{hh} = \begin{cases} 1 & \text{for inliers, i.e. with probability } (1-\epsilon) \\ \lambda^2 & \text{for outliers, i.e. with probability } \epsilon. \end{cases}$$

eq.(41)

# Kalman - LEL

$$\text{CEE\%}(k) = \frac{100}{k}\sum_{i=1}^{k}\frac{\|\hat{\mathbf{x}}_i^{+} - \mathbf{x}_i\|}{\|\mathbf{x}_i\|}. \qquad \overline{\text{CEE\%}}(k) = \frac{1}{M}\sum_{l=1}^{M}\text{CEE\%}_l(k).$$

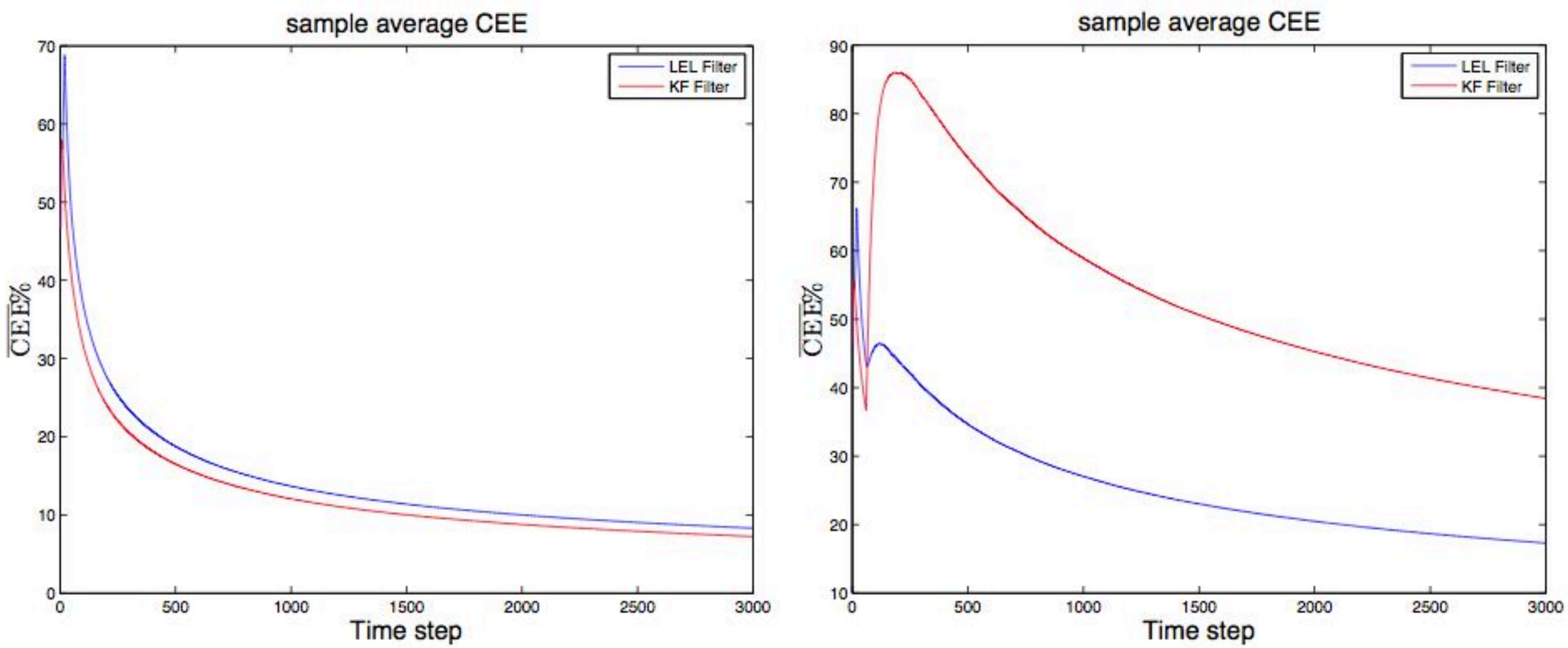



Figure 1. $\overline{\text{CEE}}$ comparison of Kalman filter and robust LEL filter in the case of: (left) Gaussian noise ($\epsilon = 0$) and (right) contaminated Gaussian noise ($\epsilon = 0.20$), from 3000 Monte Carlo runs assuming outliers generated through the model in equation (41).

# Kalman - LEL

$$\mathbf{x}_k = A_{k-1}\mathbf{x}_{k-1} + B_{k-1}\mathbf{u}_{k-1} + \mathbf{w}_{k-1}$$

State equation
zero mean
gaussian noise
covariance

$$\mathbf{y}_k = C_k\mathbf{x}_k + \mathbf{v}_k$$

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad Q = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

$$\mathbf{y}_k = \Gamma_k C \mathbf{x}_k + \mathbf{v}_k, \quad \mathbf{v}_k \sim \mathcal{N}(0, I_{3\times 3})$$

$$(\Gamma_k)_{ii} = \begin{cases} 1 & \text{for inliers} \\ \mu & \text{for outliers.} \end{cases}$$

eq.(42)

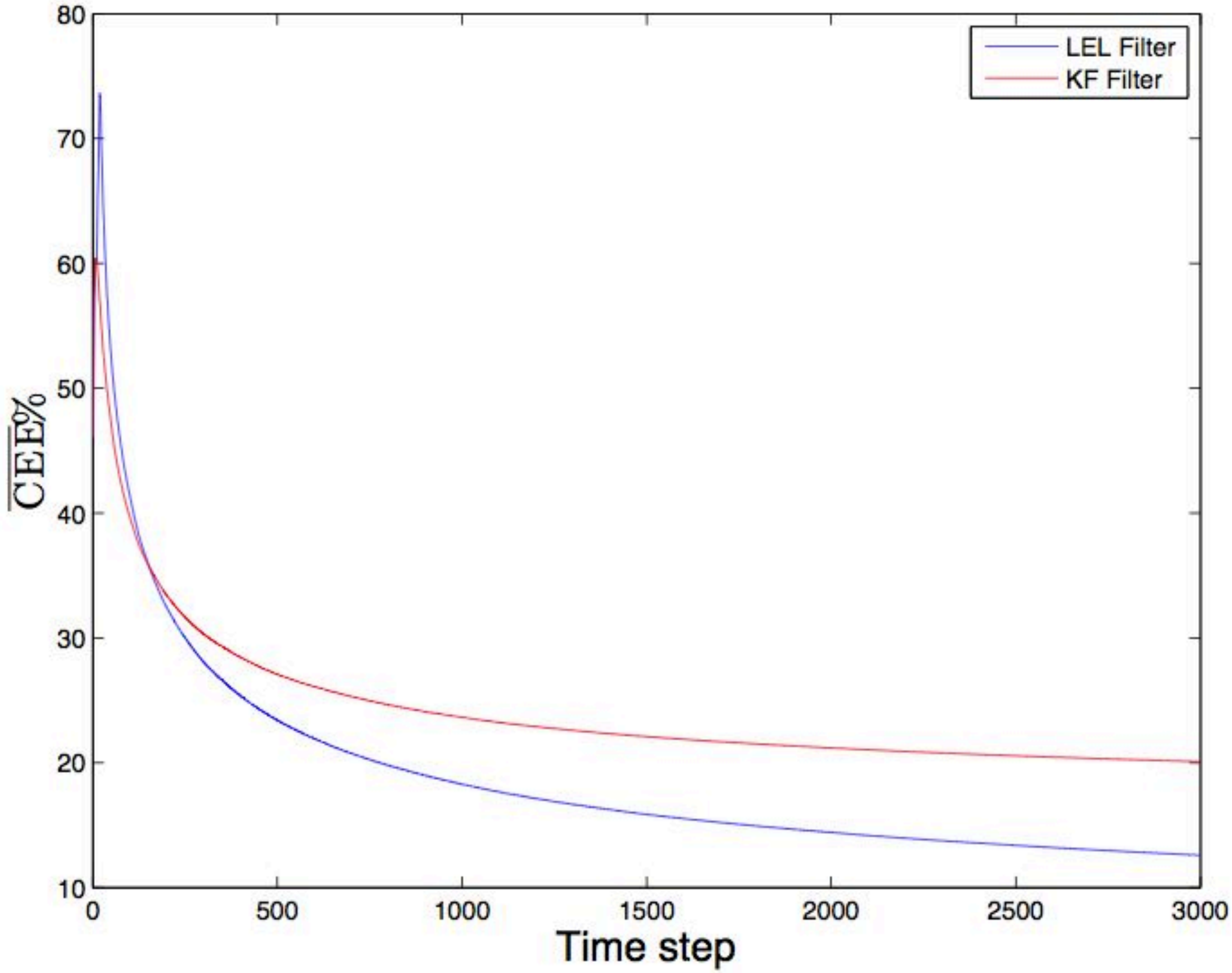



Figure 8. $\overline{\mathrm{CEE}}$ comparison of Kalman filter and robust LEL filter over 3000 Monte Carlo runs in the case of contaminated Gaussian noise ($\epsilon = 0.05$, $\mu = 3$) assuming outliers generated through the model in equation (42).

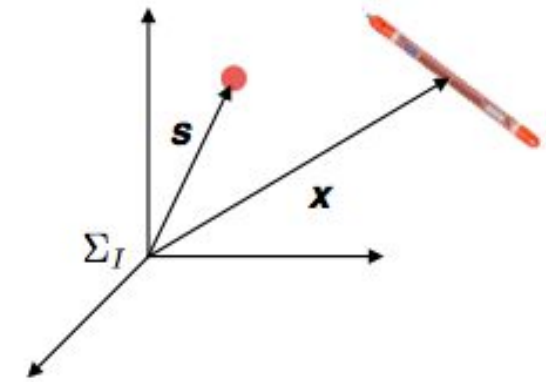
Original nonlinear model

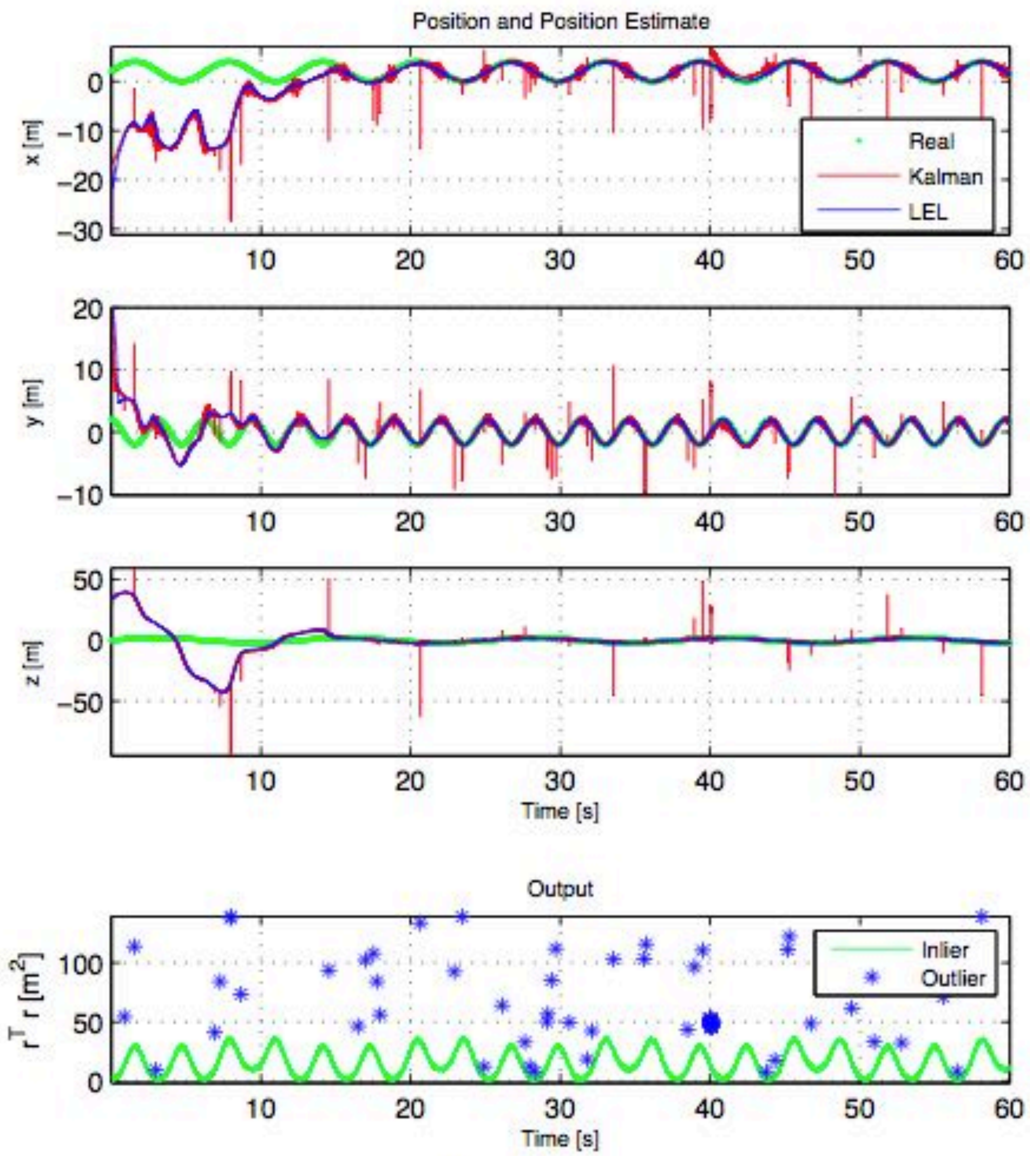
$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{v}_r + \mathbf{v}_f \\ \dot{\mathbf{v}}_f &= \mathbf{0} \\ \dot{\mathbf{s}} &= \mathbf{0} \\ y &= \|\mathbf{s} - \mathbf{x}\|^2 \end{aligned}$$

$\Longrightarrow$

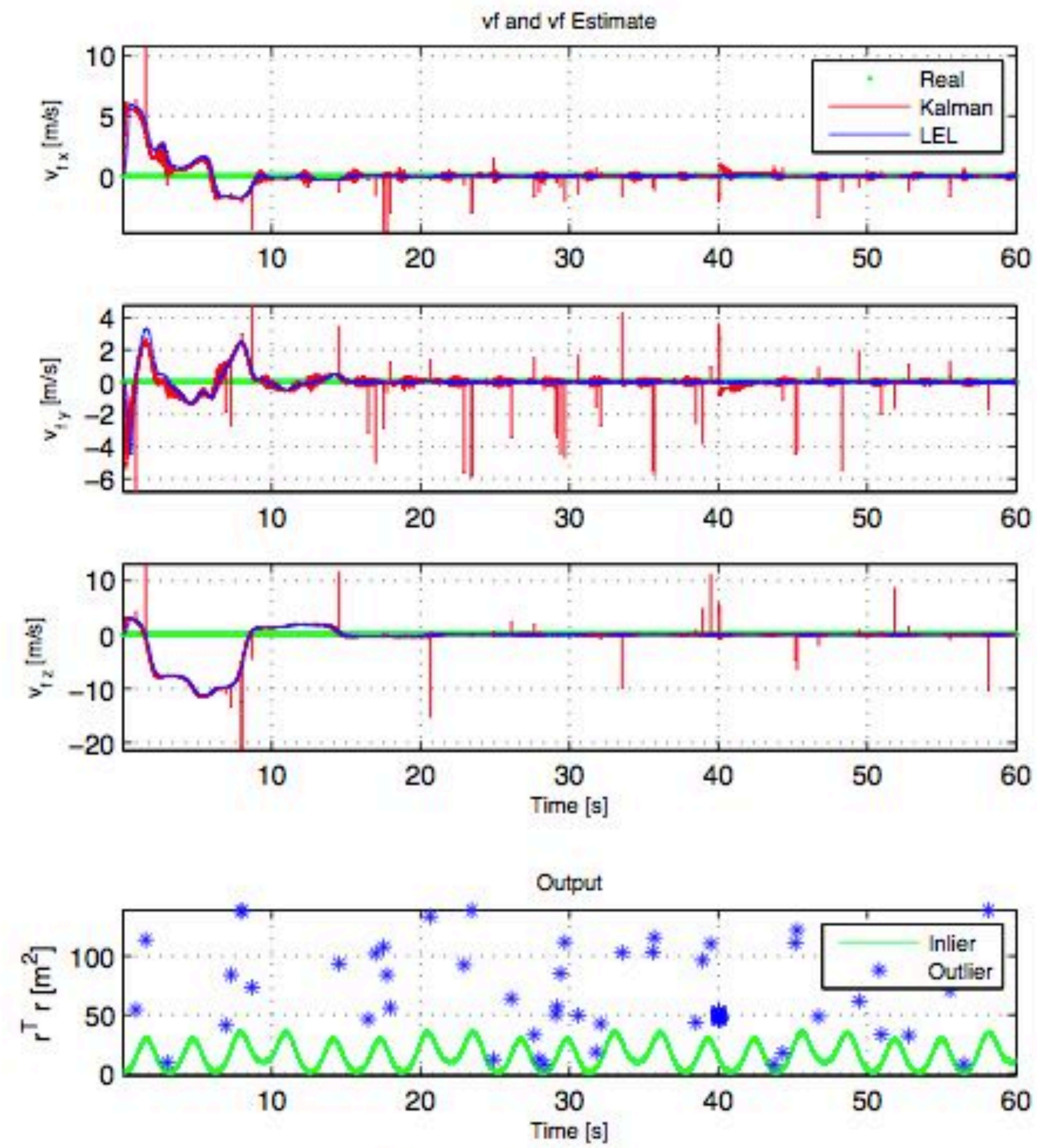
Linear model

$$\begin{aligned} \dot{\mathbf{z}} &= A\mathbf{z} + B\mathbf{v}_r \\ \bar{y}(t) &= C(t)\,\mathbf{z} \end{aligned}$$



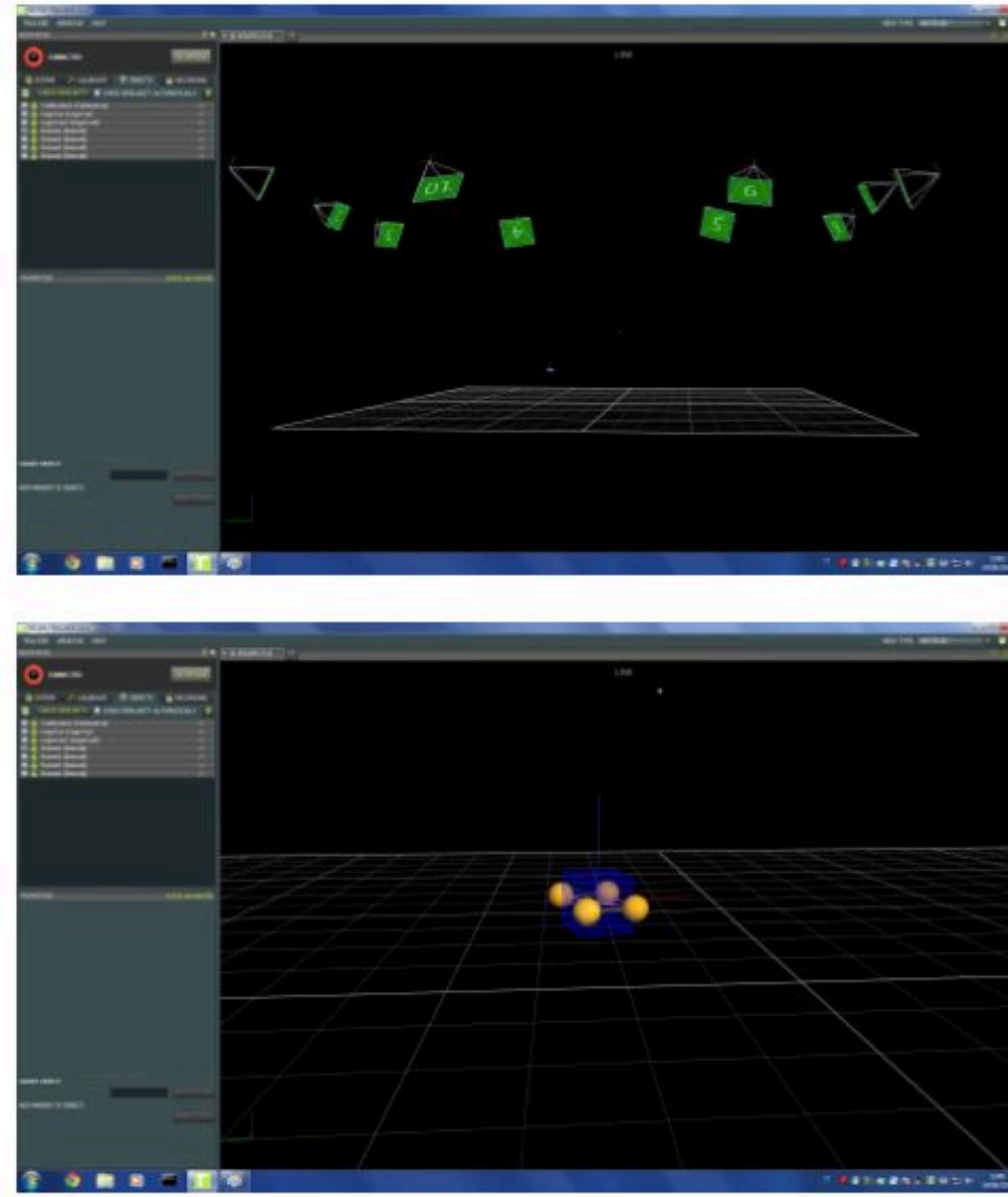




Position Estimation




Ocean Current Estimation

Vicon's B10 camera



Vicon Tracker application

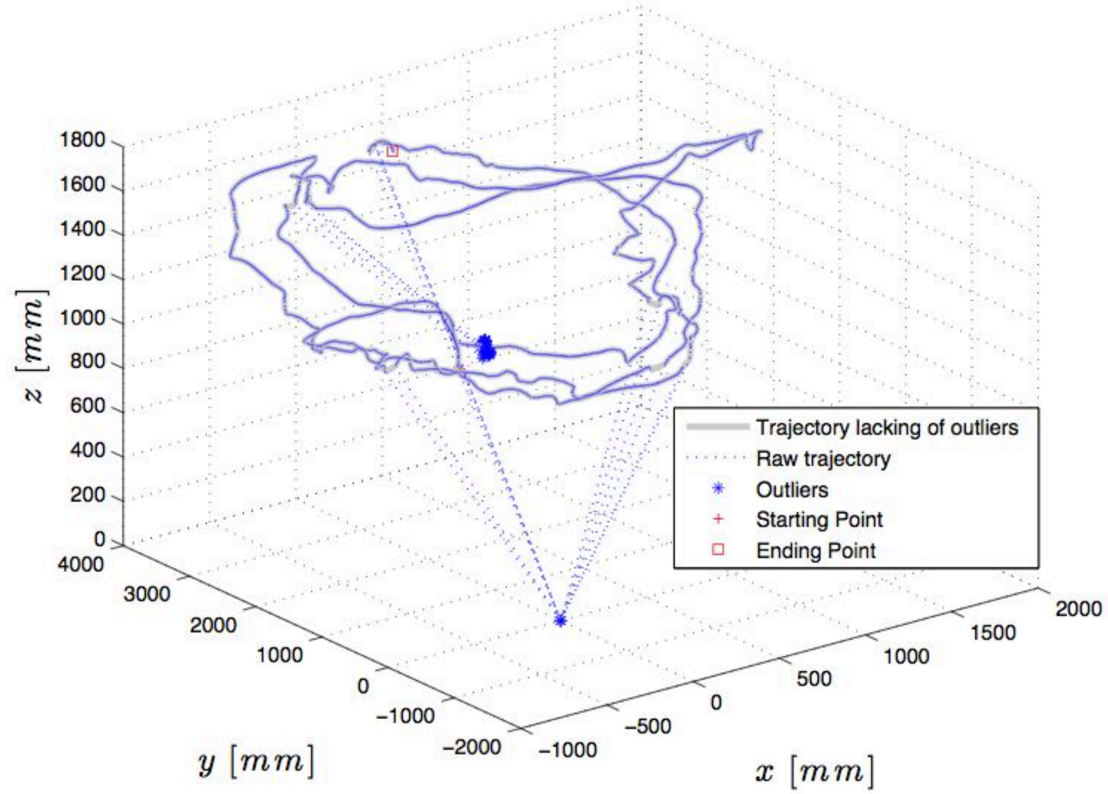
# System model

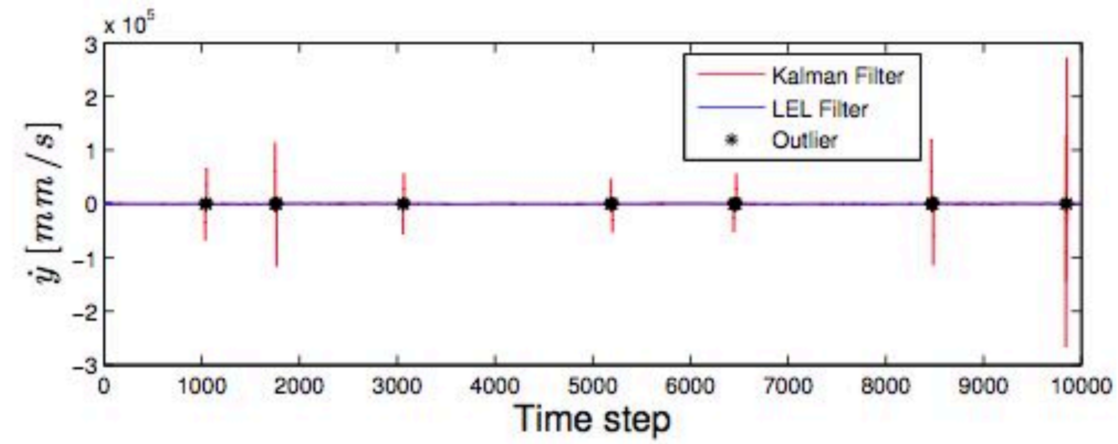
$$A = \begin{bmatrix} I_{3\times3} & I_{3\times3}\,\Delta T & I_{3\times3}\,\Delta T^2/2 \\ 0_{3\times3} & I_{3\times3} & I_{3\times3}\,\Delta T \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \quad B = \begin{bmatrix} 0_{3\times1} \\ 0_{3\times1} \\ 0_{3\times1} \end{bmatrix}$$

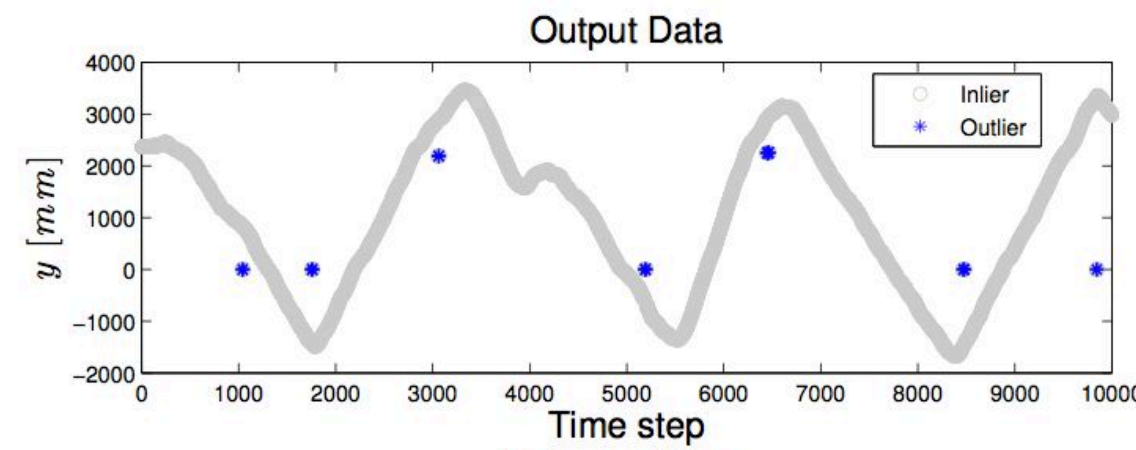
$$C = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix}$$

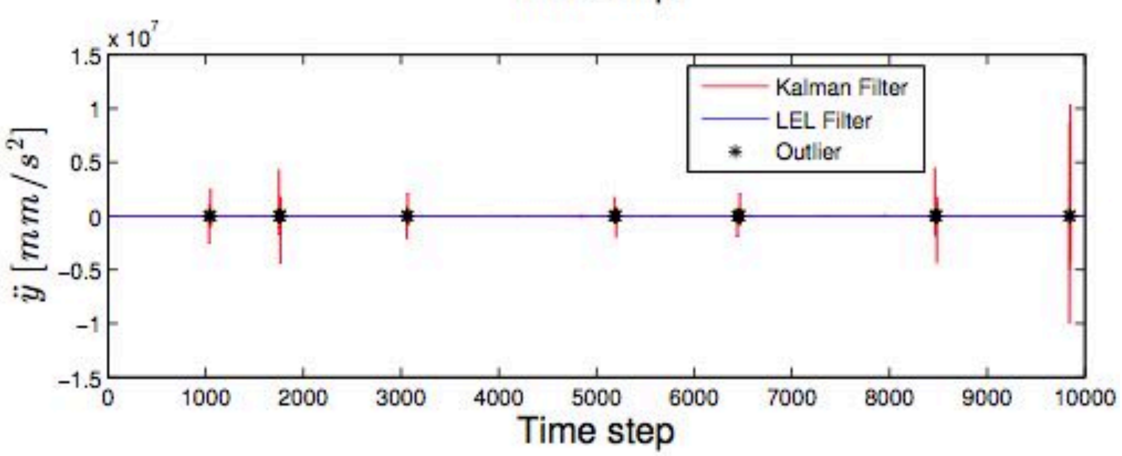
$$Q = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3}/\Delta T^2 & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3}/\Delta T^4 \end{bmatrix} \quad R = I_{3\times3},$$

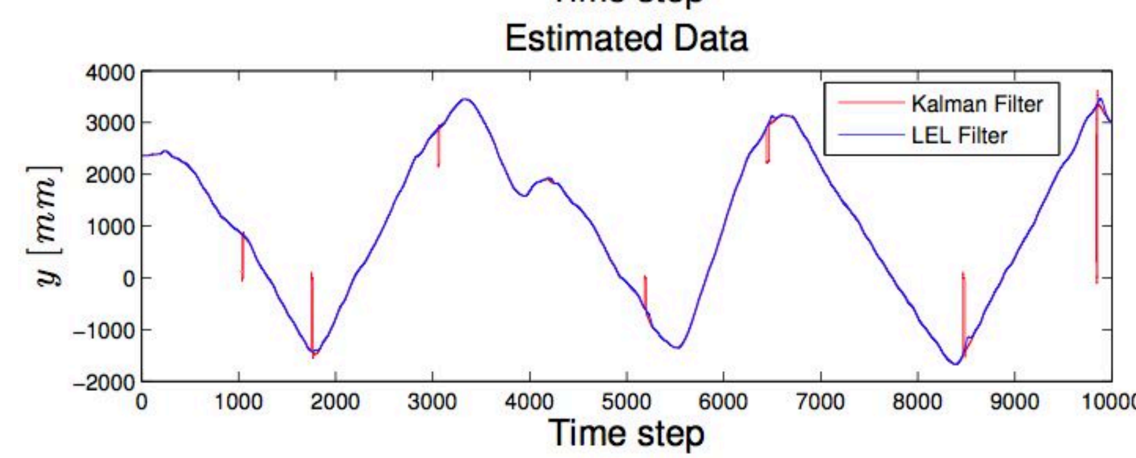
$$\Delta T = 0.01s$$



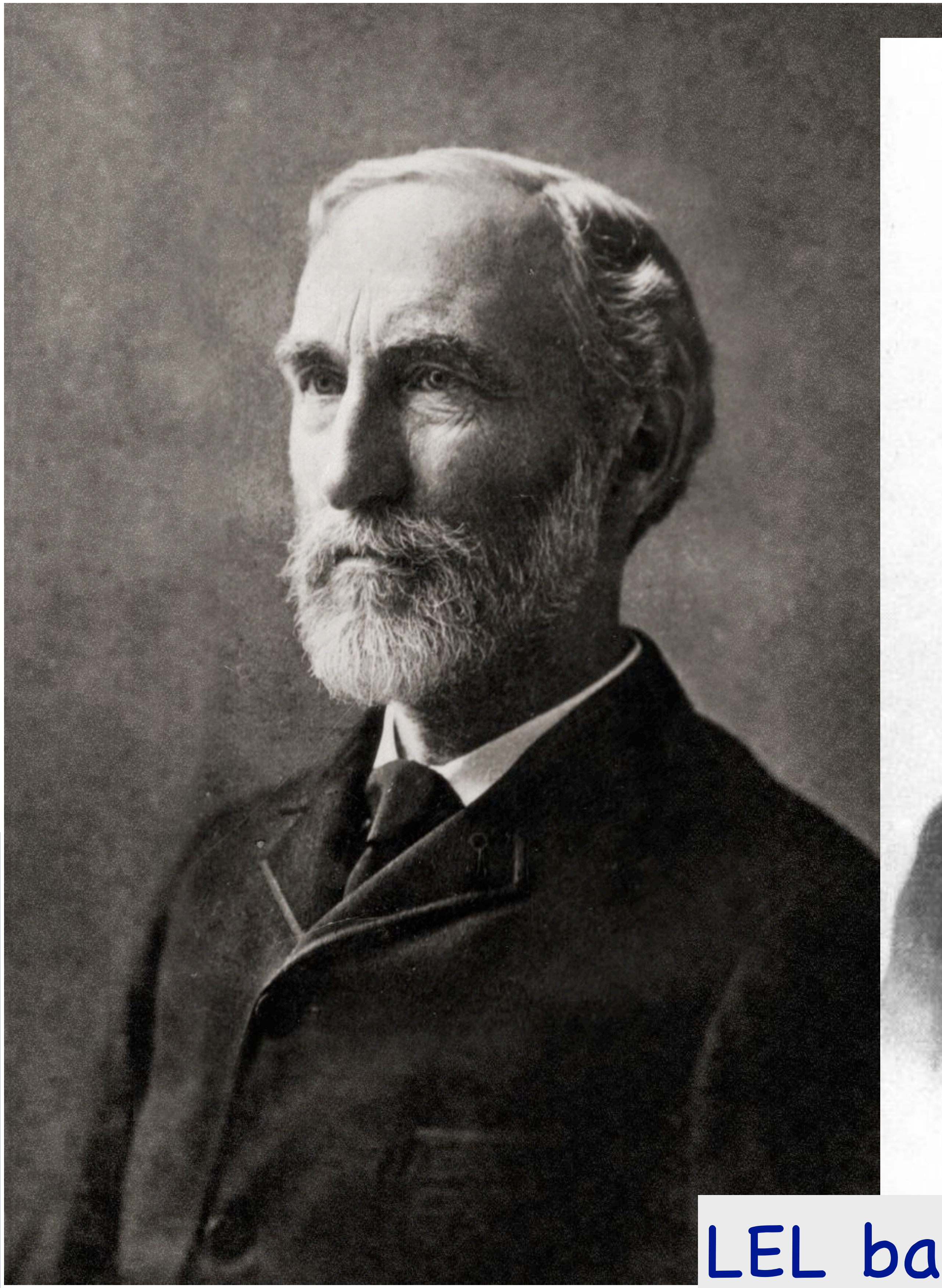



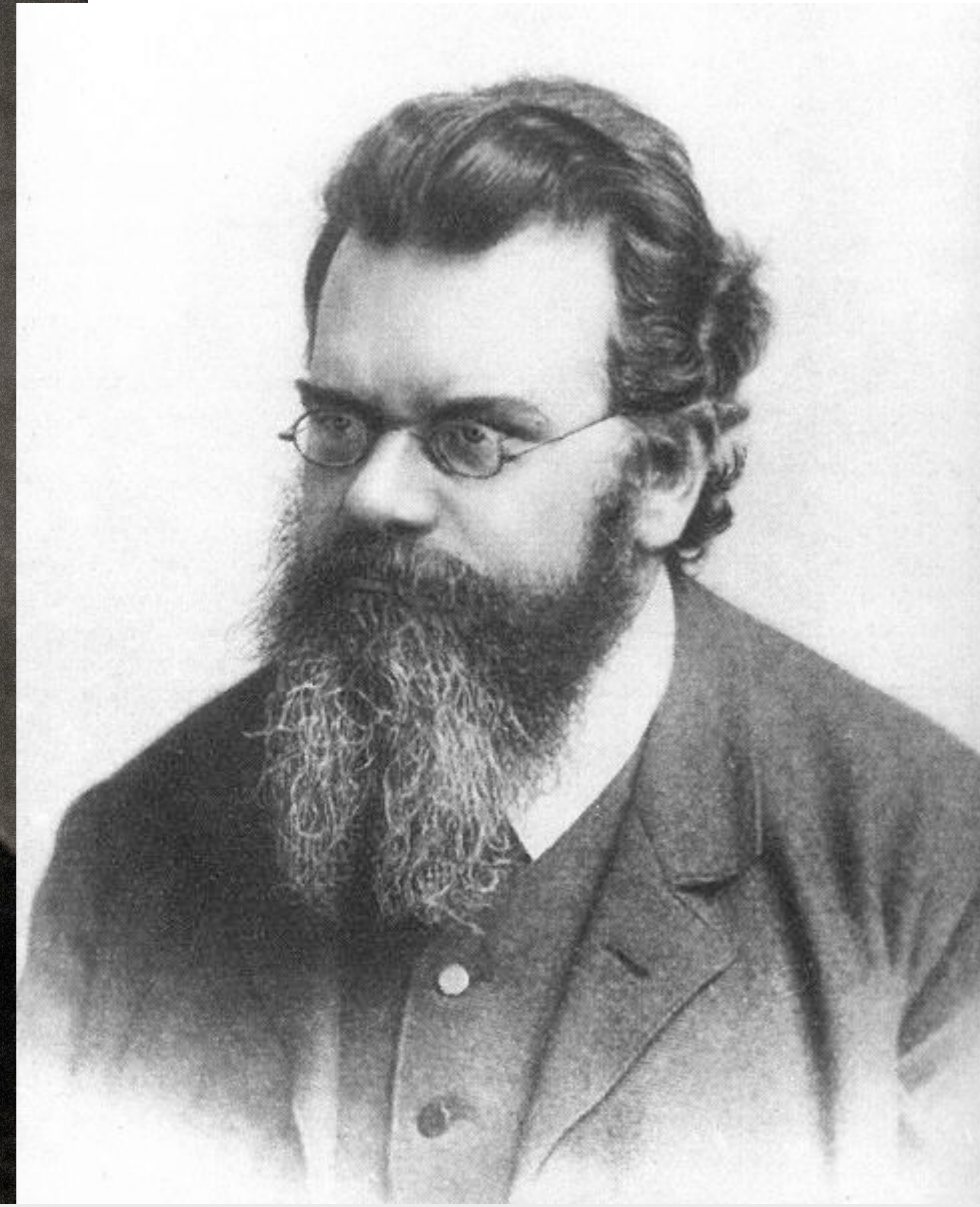



# Conclusions



Marine robotics: yet some work to be done





LEL based filtering: it works!

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# Thank you!

Question Time

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